JOHN TEMPLETON FOUNDATION
SUPPORTING SCIENCE~INVESTING IN THE BIG QUESTIONS
Project: *A Quantum-Digital Universe*, Grant ID: 43796

Quantum walks on Cayley graphs: theoretical physics and geometric group theory

Giacomo Mauro D'Ariano
Università degli Studi di Pavia

Topological and Homological Methods in Group Theory
Faculty of Mathematics, Bielefeld, April 4-8, 2016

Geometric group theory: *summa theologica*

Goal of GGT is to study finitely-generated (f.g.) groups G as automorphism groups (symmetry groups) of physical theory*.*

<u>Central question</u>: How algebraic properties of a group G reflect in dynamical *properties of a* physical theory *and, conversely, how* dynamics of a physical theory *reflects in algebraic structure of G.*

This interaction between groups and physics theory *is a fruitful 2-way road.*

From *Lectures on quasi-isometric rigidity,*
by Michael Kapovich

Program

To derive the whole Physics axiomatically

from “principles” stated in form of purely mathematical axioms (without “physical primitives”),
but having a thorough physical interpretation.

Solution: informationalism

The sixth Hilbert problem

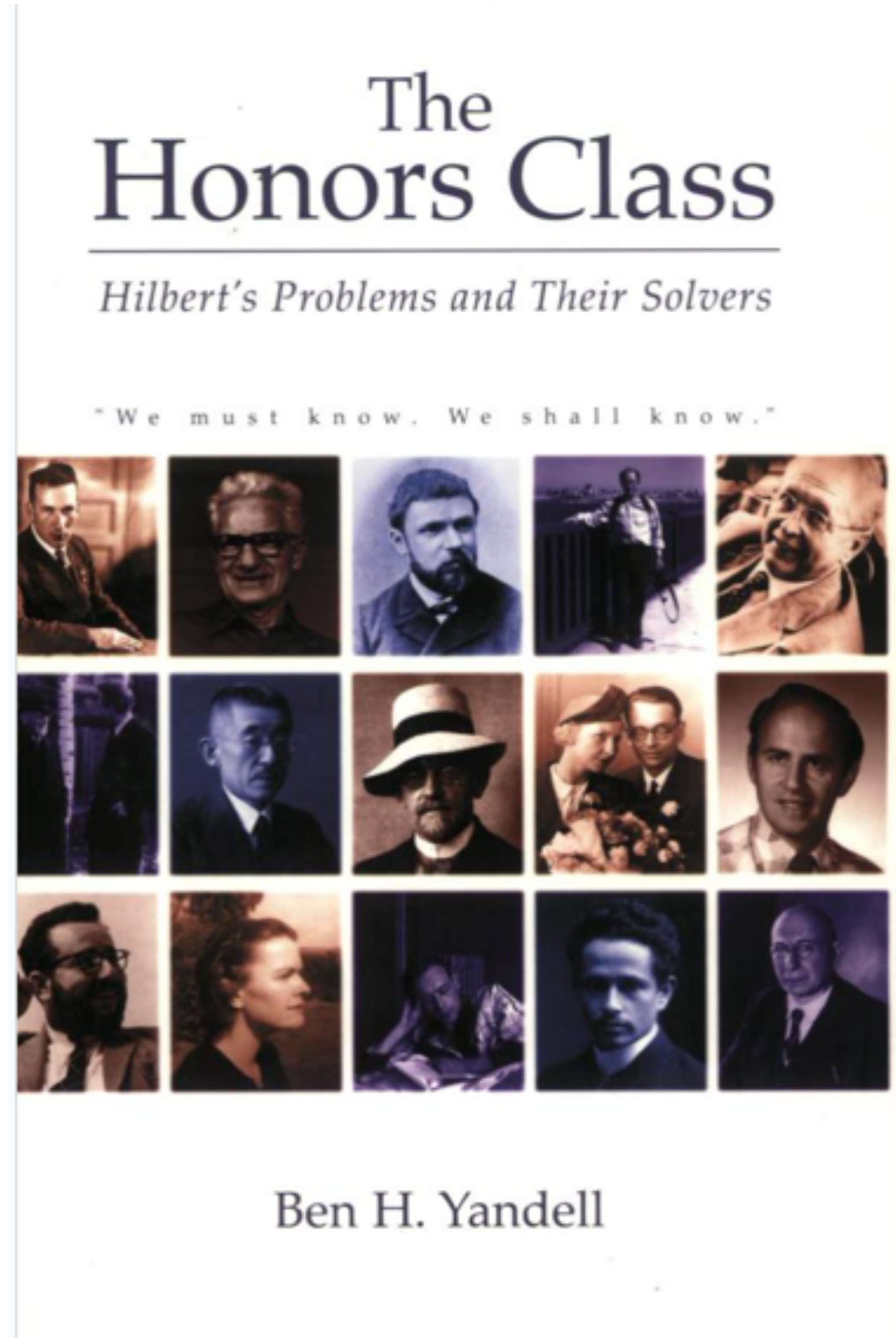
The investigations on the foundations of geometry suggest the problem: To treat in the same manner by means of axioms, those physical sciences in which mathematics plays an important part; in the first rank are the theory of probabilities and mechanics.

David Hilbert

Mechanics: the Trojan horse

Axiomatizing the theory of probabilities was a realistic goal: Kolmogorov accomplished this in 1933. The word 'mechanics' without a qualifier, however, is a Trojan horse."

Benjamin Yandell



Principles for Quantum Theory

Selected for a Viewpoint in *Physics*

PHYSICAL REVIEW A **84**, 012311 (2011)

Informational derivation of quantum theory

Giulio Chiribella*
Perimeter Institute for Theoretical Physics, 31 Caroline Street North, Ontario, Canada N2L 2Y5†

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(Received 29 November 2010; published 11 July 2011)

We derive quantum theory from purely informational principles. Five elementary axioms—causality, perfect distinguishability, ideal compression, local distinguishability, and pure conditioning—define a broad class of theories of information processing that can be regarded as standard. One postulate—purification—singles out quantum theory within this class.

DOI: 10.1103/PhysRevA.84.012311 PACS number(s): 03.67.Ac, 03.65.Ta

Principles for Quantum Theory

P1. Causality

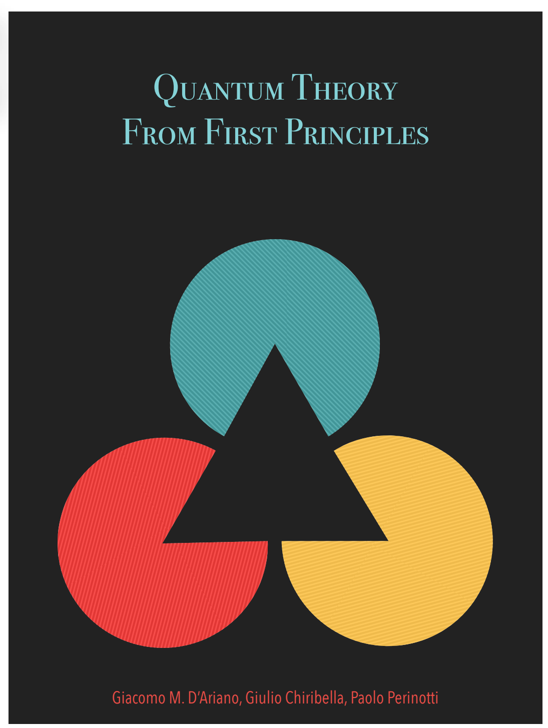
P2. Local discriminability

P3. Purification

P4. Atomicity of composition

P5. Perfect distinguishability

P6. Lossless Compressibility



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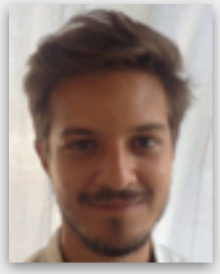
Principles for Mechanics


Paolo Perinotti


Alessandro Bisio


Alessandro Tosini


Marco Erba


Franco Manessi


Nicola Mosco

- *Mechanics (QFT) derived in terms of countably many quantum systems in interaction*

add principles

Min algorithmic complexity principle

- homogeneity
- locality
- isotropy

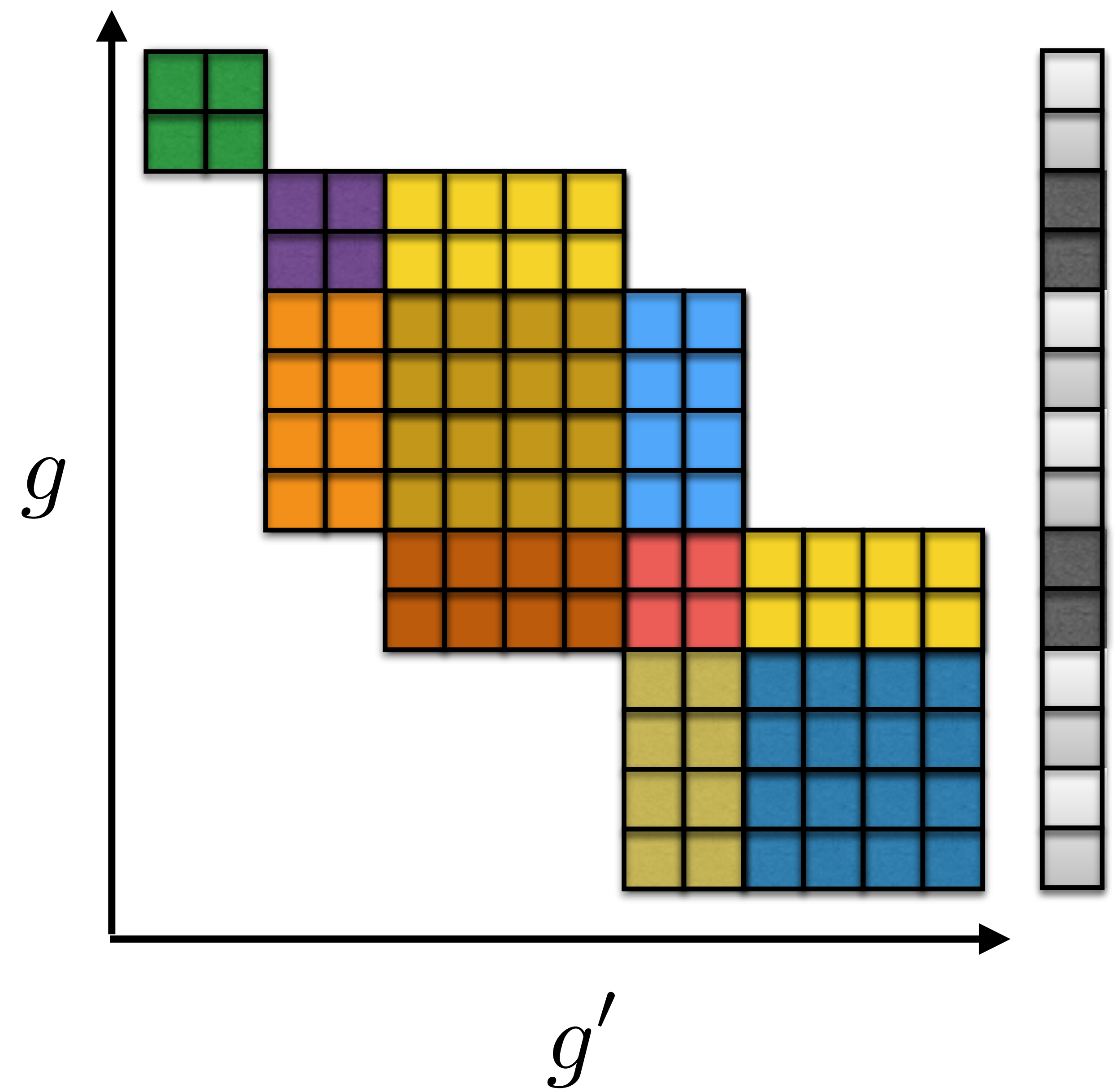
Quantum walk on Cayley graph

w.l.g. Hilbert space $\mathcal{H} = \bigoplus_{g\in G} \mathbb{C}^{s_g} \quad |G| \leqslant \aleph, \ s_g \in \mathbb{N}$

Evolution

$$\psi_g(t+1) = \sum_{g'\in S_g} A_{gg'}\psi_{g'}(t)$$

$$\sum_{g'} A_{gg'}A^\dagger_{g''g'} = \sum_{g'} A^\dagger_{gg'}A_{g''g'} = \delta_{gg''}I_{s_g}$$



D'Ariano, Perinotti,
PRA **90** 062106 (2014)

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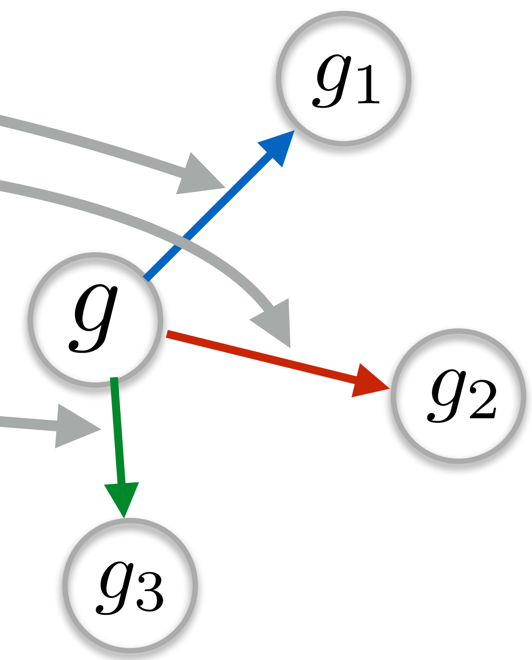
Build a <u>directed graph</u> with an arrow from *g* to *g' wherever they* are connected by $A_{gg'} \neq 0$



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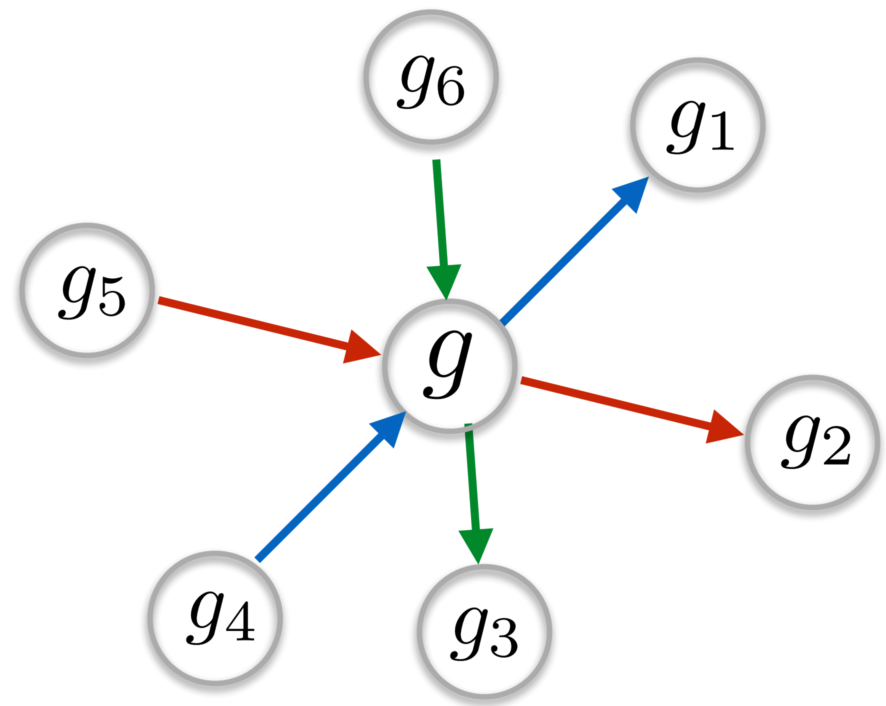

1) *Locality*: S_g uniformly bounded

2) *Reciprocity*: $A_{gg'} \neq 0 \implies A_{g'g} \neq 0$

3) *Homogeneity*: all $g \in G$ are "equivalent"

D'Ariano, Perinotti,
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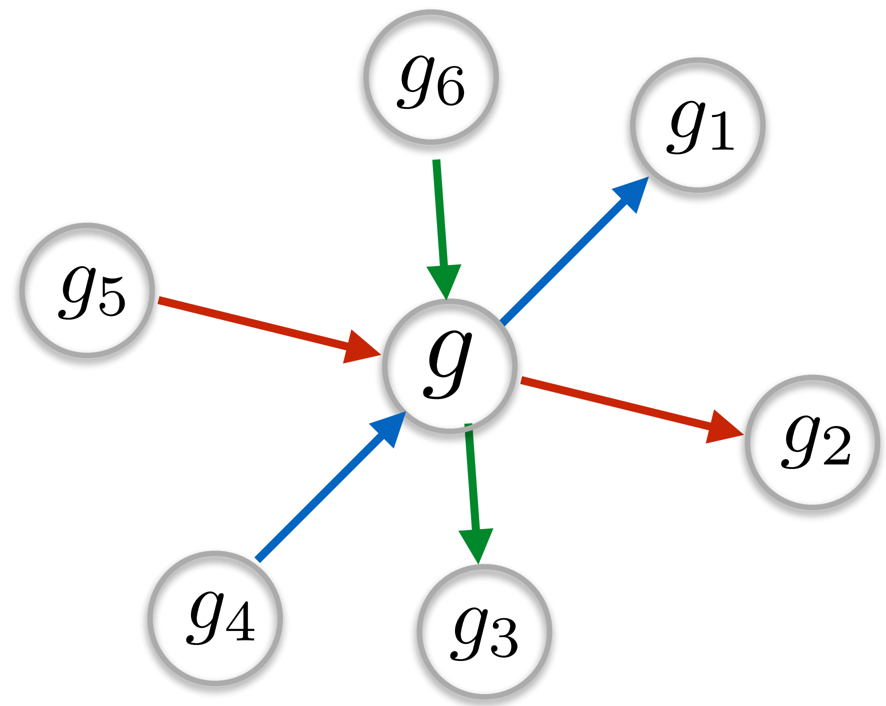

1) Locality: S_g uniformly bounded

2) Reciprocity: $A_{gg'} \neq 0 \implies A_{g'g} \neq 0$

3) Homogeneity: all $g \in G$ are "equivalent"

$S_g = S, \; s_g = s \; \ldots$ label $A_{gg'} =: A_h, \; h \in: S$

define the "action" on the set of vertices *G*: $gh := g'$ whenever $A_{gg'} = A_h$

D'Ariano, Perinotti,
PRA **90** 062106 (2014)

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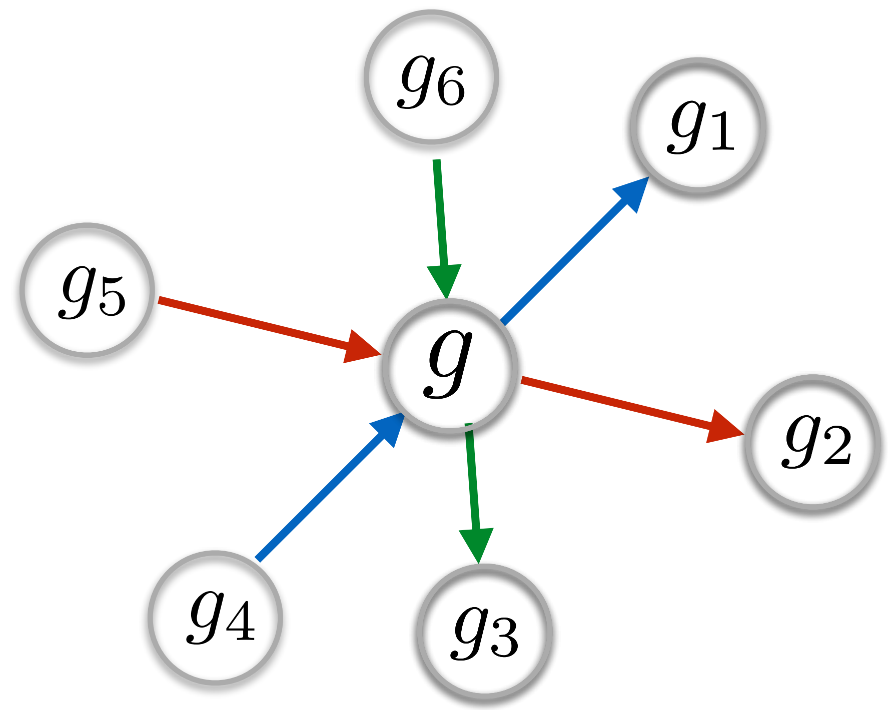

1) *Locality*: S_g uniformly bounded

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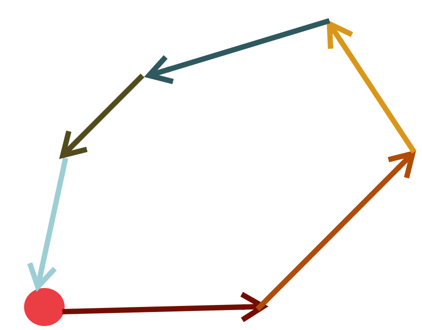
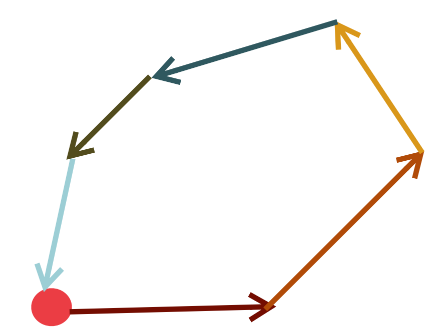
3) *Homogeneity*: all $g \in G$ are equivalent

A sequence $A_{h_N}A_{h_{N-1}}\ldots A_{h_1}$ connects g to itself, namely $gh_1h_2\ldots h_N = g$, then it must also connect any other g' to itself, i.e. $g'h_1h_2\ldots h_N = g'$.





From 2): two-loop $ghh^{-1} = g$ defines uniquely h^{-1} for h and viceversa

$$A_{gg'} =: A_h,\ A_{g'g} =: A_{h^{-1}},\ h \in S \equiv S_+ \cup S_-,\ S_- := S_+^{-1}$$

D'Ariano, Perinotti, PRA **90** 062106 (2014)

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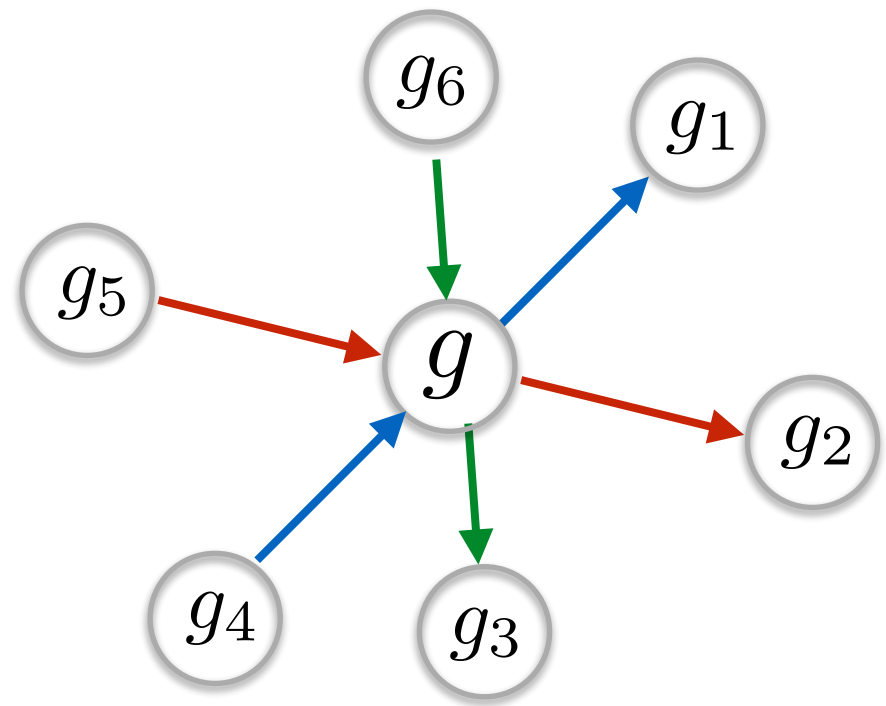

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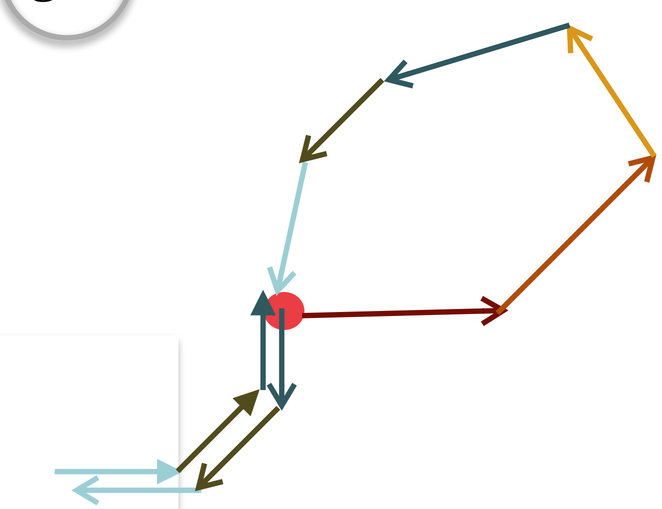

Build the free group *F* of words made with letters:

$$h \in S := S_+ \cup S_-$$

with action on vertices in *G*: $gh := g'$ whenever $A_{gg'} = A_h$

Consider the subgroup *H* of closed paths ➡ *H* normal subgroup of *F*

D'Ariano, Perinotti, PRA **90** 062106 (2014)

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$\Gamma(G, S_+)$ *colored directed graph* with vertices $g \in G$ and edges (g, g') with $g' = gh$

Either the graph is connected, or it consists of disconnected copies.
W.l.g. assume it as connected.

Being H normal, one concludes that:

$G = F/H = \langle S|R\rangle$ is a group with Cayley graph $\Gamma(G, S_+)$ (the identity any element $e \in G$).

D'Ariano, Perinotti,
PRA **90** 062106 (2014)

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$\Big\}$ iff for Quantum Walk on Cayley graph

D'Ariano, Perinotti,
PRA **90** 062106 (2014)

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The following operator over the Hilbert space $\ell^2(G)\otimes\mathbb{C}^s$ is unitary

$$A = \sum_{h\in S} T_h \otimes A_h$$

where T is the right regular representation of G on $\ell^2(G)$ acting as

$$T_g|g'\rangle = |g'g^{-1}\rangle$$

1) *Locality*: S_g uniformly bounded

2) *Reciprocity*: $A_{gg'} \neq 0 \implies A_{g'g} \neq 0$

3) *Homogeneity*: all $g \in G$ are equivalent

4) *Isotropy:*

There exist:

- a group L of permutations of S_+, transitive over S_+ that leaves the Cayley graph invariant
- a unitary s-dimensional (projective) representation $\{L_l\}$ of L such that:

$$A = \sum_{h\in S} T_h \otimes A_h = \sum_{h\in S} T_{lh}\otimes L_l A_h L_l^\dagger$$

Quantum walk on Cayley graph

The quantum walk on the Cayley graph (QWCG) is completely specified as

$$Q = (G, S_+, s, \{A_h\}_{h \in S})$$

In the following we will restrict to Cayley graphs <u>qi-embeddable in *R^d*</u>

- **Thm.** [Misha Kapovich] *G* is a finitely-generated group whose Cayley graph qi embeds in *R^d* *iff G* contains a free Abelian subgroup *H* of finite index, with *rank(A)=d*.
- **Proof.** *R^d* has polynomial growth, equivalent to *x^d*. Thus, *G* also has growth at most *x^d*. By Gromov's theorem, it follows that *G* is virtually nilpotent. For nilpotent groups there is a precise formula for growth in terms of their derived series [Bass and Guivarch] which implies that the group has to be virtually Abelian of rank ≤d.

Quantum walk on Cayley graph

The quantum walk on the Cayley graph (QWCG) is completely specified as

$$Q = (G, S_+, s, \{A_h\}_{h \in S})$$

In the more general case restrict to H with solvable word problem and finite generating set R, i.e. G finitely presented (true for virtually Abelian).

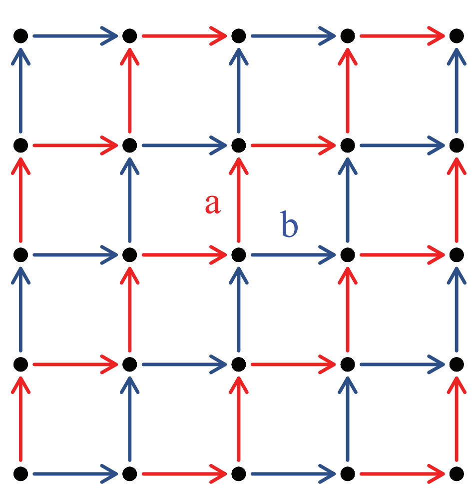
D'Ariano, Erba, Perinotti, Tosini
arXiv::1603.07666,1511.03992

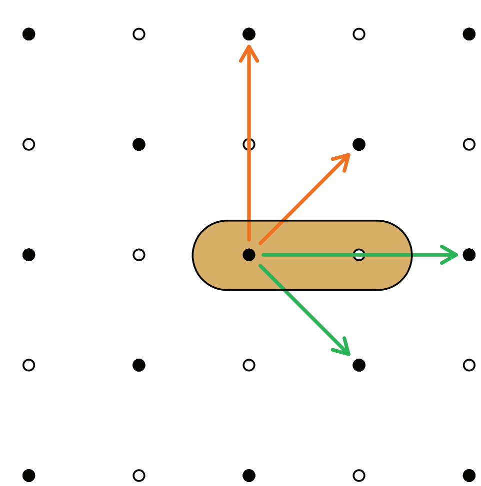
Quantum walk on Cayley graph: *"renormalization"*

A QWCG $Q = (G, S_+, s, \{A_h\}_{h\in S})$ with G virt. Abelian is equivalent to a QWCG $Q' = (H, S_+, s i_H, \{A_h\}_{h\in S})$ with $H \subset G$ with index i_H (induced representation).

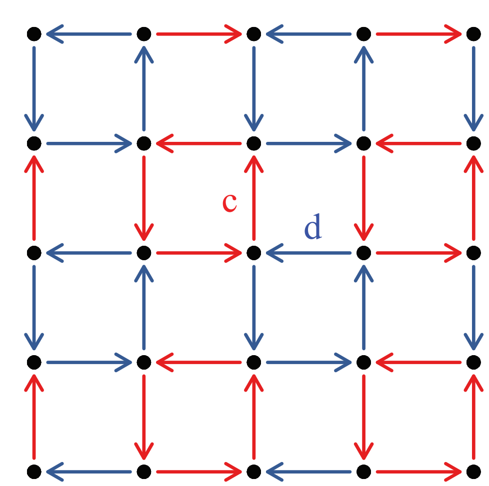
(but isotropy is not transferred between G and H)

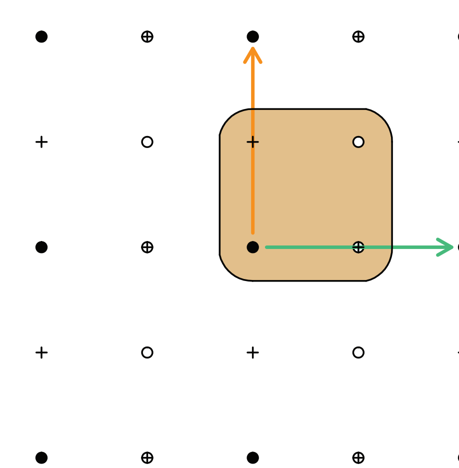
$\langle\, a, b \mid a^2 b^{-2} \,\rangle$





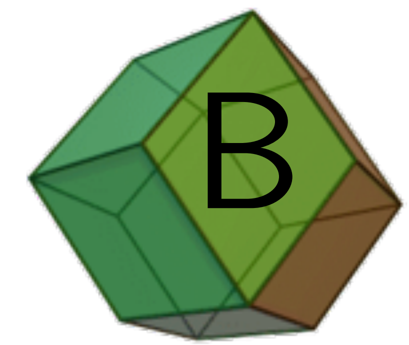
$\langle\, c, d \mid c^4, d^4, (cd)^2 \,\rangle$





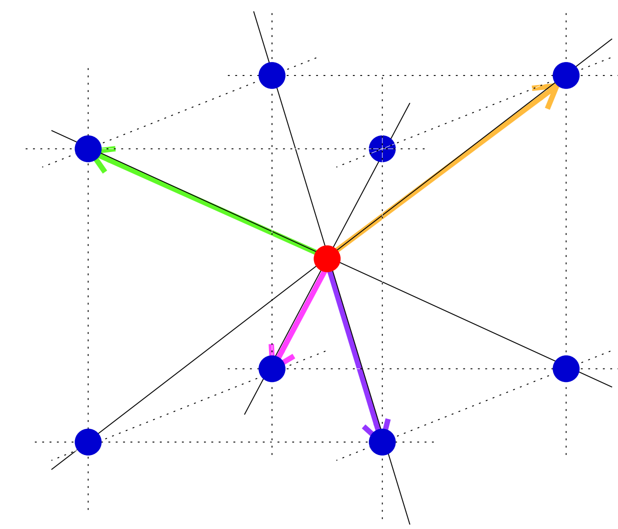
The Weyl QW

☞ Minimal dimension for nontrivial unitary *A:* s=2



- Unitarity ⇒ for d=3 the only possible *G* is the BCC!!

- Isotropy ⇒ Fermionic ψ (d=3)



Unitary operator: $$A = \int_B^{\oplus} d\mathbf{k}\, A_{\mathbf{k}}$$

Two QWs connected by P

$$\boxed{\begin{aligned} A_{\mathbf{k}}^{\pm} = & - i\sigma_x(s_x c_y c_z \pm c_x s_y s_z) \\ & \mp i\sigma_y(c_x s_y c_z \mp s_x c_y s_z) \\ & - i\sigma_z(c_x c_y s_z \pm s_x s_y c_z) \\ & + I(c_x c_y c_z \mp s_x s_y s_z) \end{aligned}}$$

$$s_\alpha = \sin\frac{k_\alpha}{\sqrt{3}}$$

$$c_\alpha = \cos\frac{k_\alpha}{\sqrt{3}}$$

The Weyl QW

D'Ariano, Perinotti,
PRA **90** 062106 (2014)

$$i\partial_t \psi(t) \simeq \tfrac{i}{2}[\psi(t+1) - \psi(t-1)] = \tfrac{i}{2}(A - A^\dagger)\psi(t)$$

$$\begin{aligned}\tfrac{i}{2}(A^{\pm}_{\mathbf{k}} - A^{\pm\dagger}_{\mathbf{k}}) = &+ \sigma_x(s_x c_y c_z \pm c_x s_y s_z)\\ &\pm \sigma_y(c_x s_y c_z \mp s_x c_y s_z)\\ &+ \sigma_z(c_x c_y s_z \pm s_x s_y c_z)\end{aligned}$$

"Hamiltonian"

$k \ll 1$

$i\partial_t \psi = \frac{1}{\sqrt{3}} \boldsymbol{\sigma}^{\pm} \cdot \mathbf{k}\, \psi$ ☜ Weyl equation!

$\boldsymbol{\sigma}^{\pm} := (\sigma_x, \pm\sigma_y, \sigma_z)$

Two QCAs connected by P

$$\begin{aligned}A^{\pm}_{\mathbf{k}} = &- i\sigma_x(s_x c_y c_z \pm c_x s_y s_z)\\ &\mp i\sigma_y(c_x s_y c_z \mp s_x c_y s_z)\\ &- i\sigma_z(c_x c_y s_z \pm s_x s_y c_z)\\ &+ I(c_x c_y c_z \mp s_x s_y s_z)\end{aligned}$$

$$s_\alpha = \sin\frac{k_\alpha}{\sqrt{3}}$$

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D'Ariano, Perinotti, PRA **90** 062106 (2014)

Dirac QW $\oplus$

Local coupling: $A_{\mathbf{k}}$ coupled with its inverse with off-diagonal identity block matrix

$$E^{\pm}_{\mathbf{k}} = \begin{pmatrix} nA^{\pm}_{\mathbf{k}} & imI \\ imI & nA^{\pm\dagger}_{\mathbf{k}} \end{pmatrix}$$

$$n^2 + m^2 = 1 \qquad n, m \in \mathbb{R}$$

$E^{\pm}_{\mathbf{k}}$ CPT-connected!

$$\omega^{E}_{\pm}(\mathbf{k}) = \cos^{-1}[n(c_x c_y c_z \mp s_x s_y s_z)]$$

Dirac in relativistic limit $\quad k \ll 1$

m: mass, $m^2 \leq 1$

n^{-1}: refraction index

± 1

m

Bisio, D'Ariano, Perinotti, Ann. Phys. **368** 177 (2016)

Maxwell QW $\otimes$

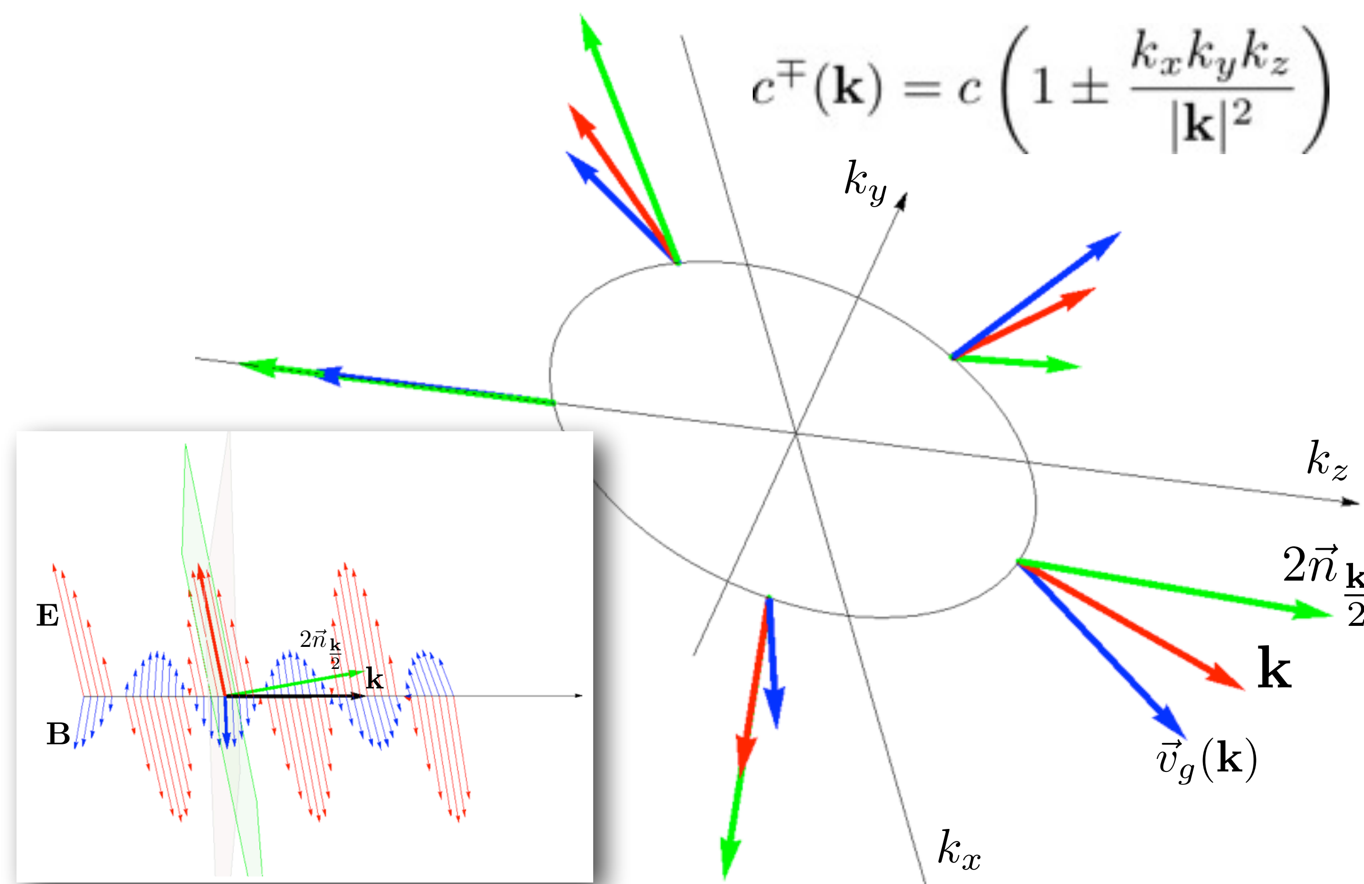
$$M^{\pm}_{\mathbf{k}} = A^{\pm}_{\mathbf{k}} \otimes A^{\pm *}_{\mathbf{k}}$$

$$F^{\mu}(\mathbf{k}) = \int \frac{\mathrm{d}\,\mathbf{q}}{2\pi} f(\mathbf{q}) \tilde{\psi}(\tfrac{\mathbf{k}}{2} - \mathbf{q}) \sigma^{\mu} \varphi(\tfrac{\mathbf{k}}{2} + \mathbf{q})$$

Maxwell in relativistic limit $\quad k \ll 1$

Boson: emergent from convolution of fermions

(De Broglie neutrino-theory of photon)



The LTM standards of the theory

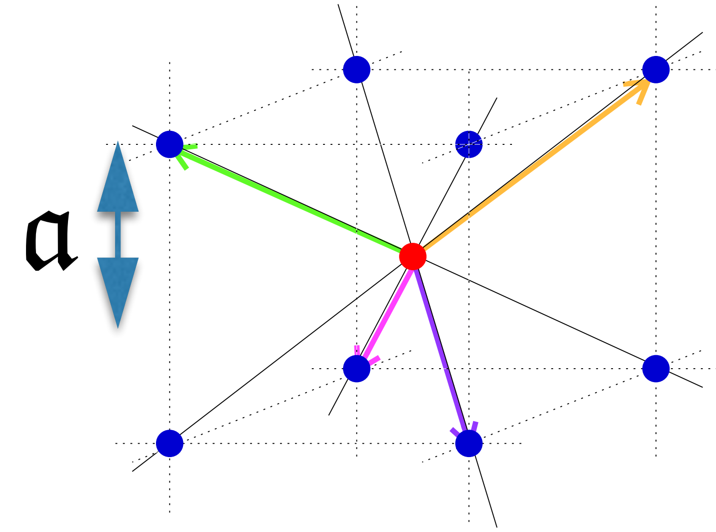
Dimensionless variables

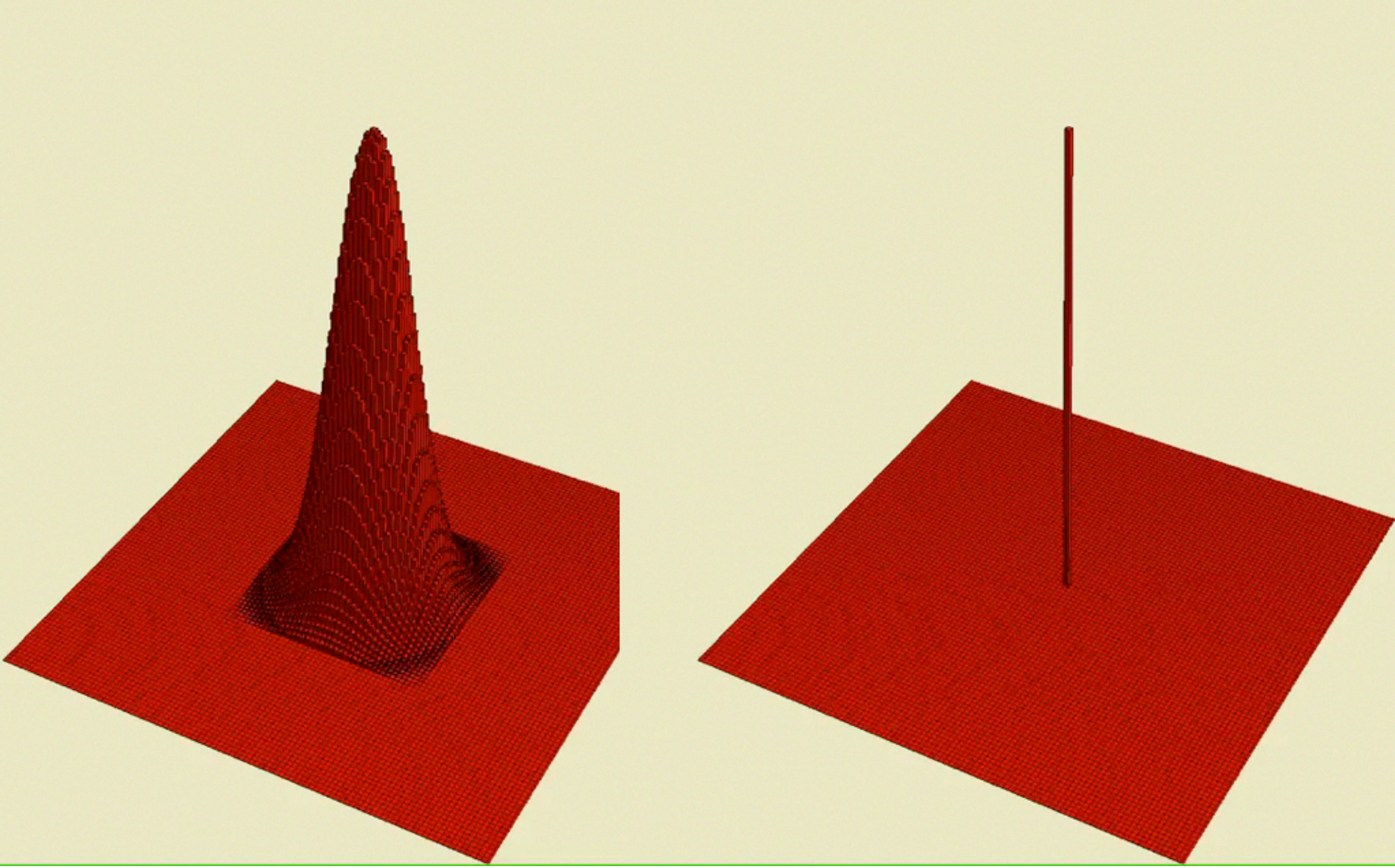
$$x = \frac{x_{[m]}}{\mathfrak{a}} \in \mathbb{Z}, \quad t = \frac{t_{[sec]}}{\mathfrak{t}} \in \mathbb{N}, \quad m = \frac{m_{[kg]}}{\mathfrak{m}} \in [0,1]$$

Relativistic limit: $\rightarrow$ $c = \mathfrak{a}/\mathfrak{t} \qquad \hbar = \mathfrak{m}\mathfrak{a}c$

Measure $\mathfrak{a}$ from light-refraction-index

$$\rightarrow \quad c^{\mp}(k) = c\left(1 \pm \frac{k}{\sqrt{3}k_{max}}\right)$$

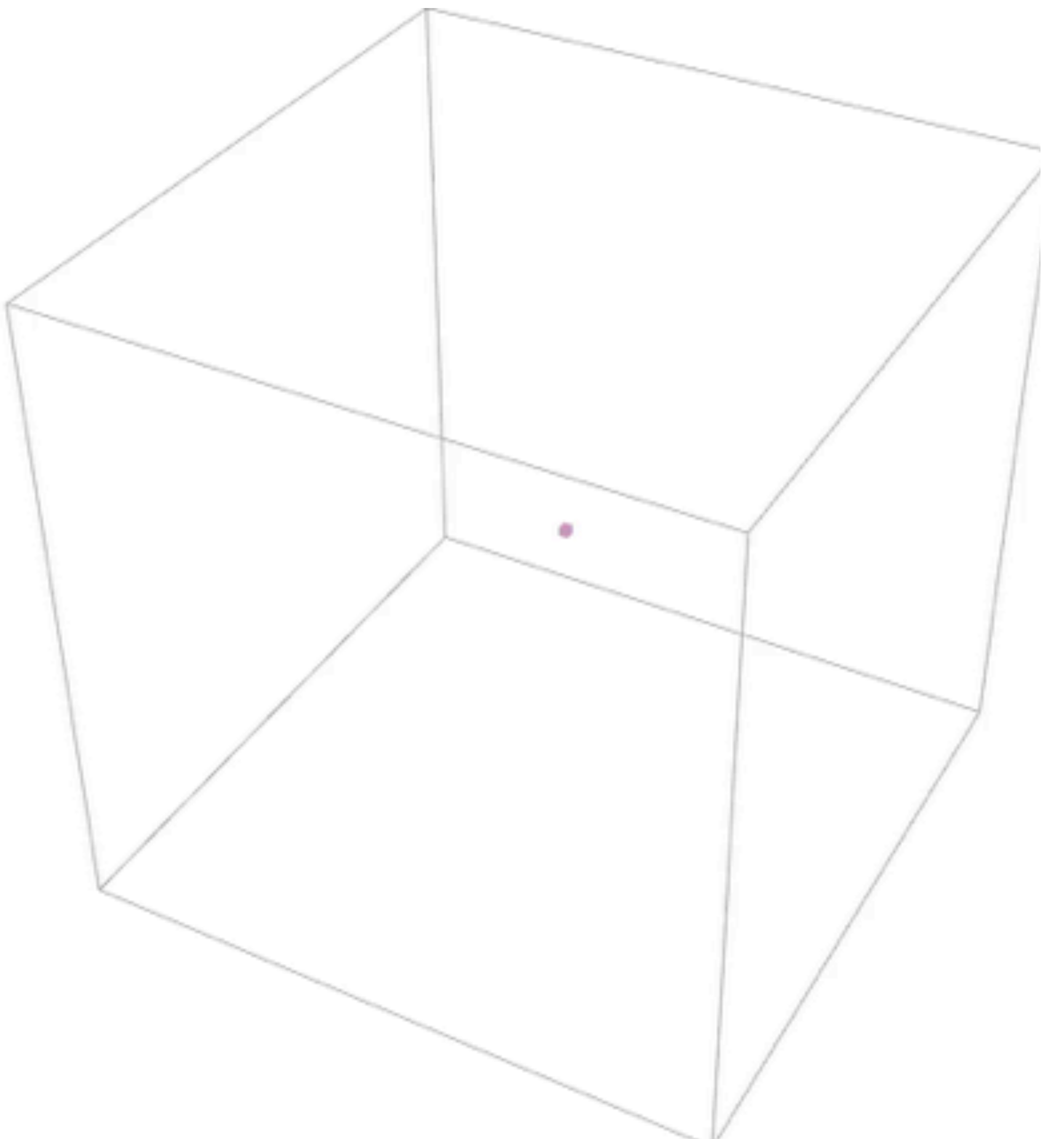


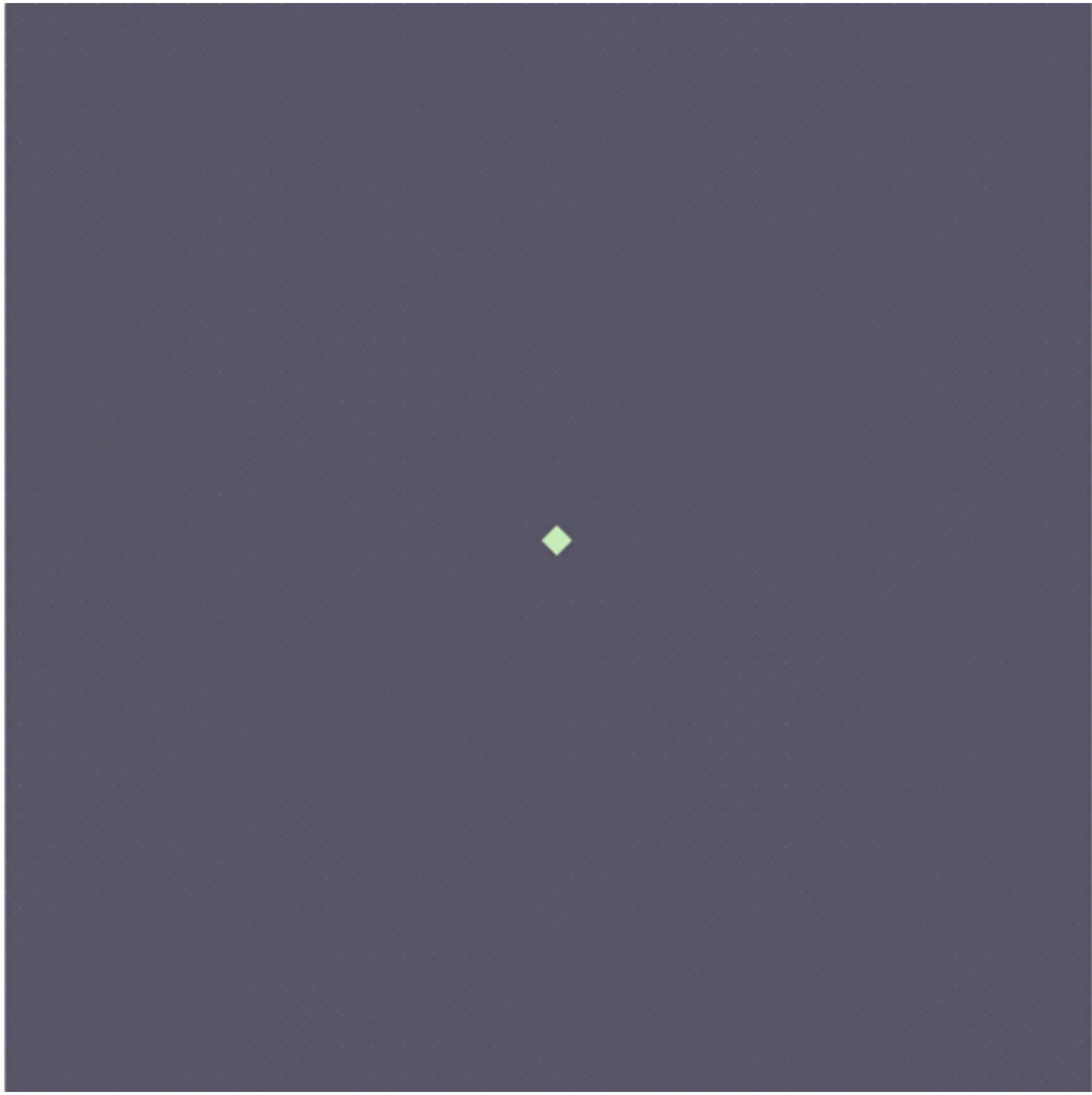


2d Dirac

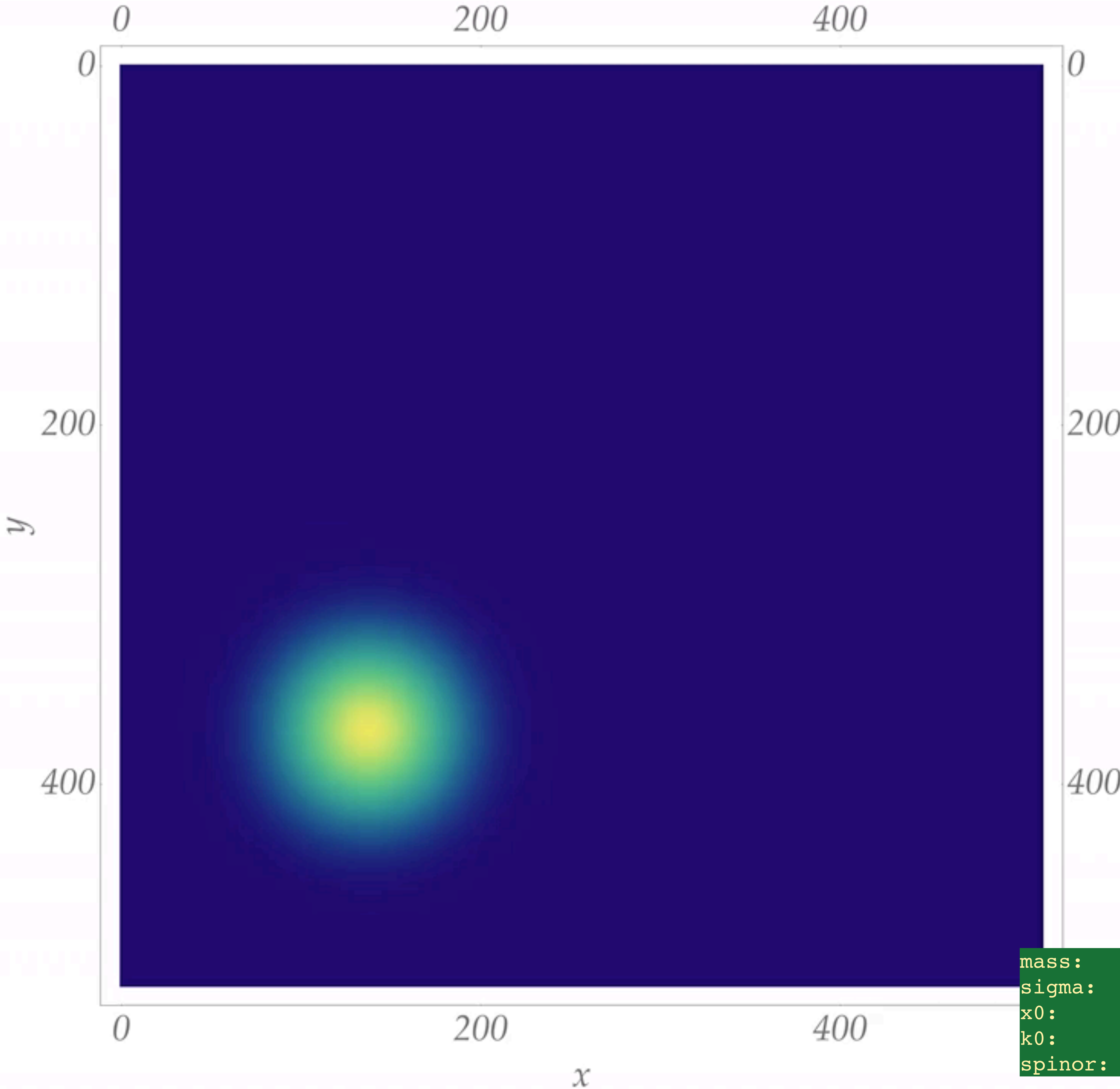
- Evolution of a *narrow-band particle-state*
- Evolution of a *localized state*

D’Ariano, Mosco, Perinotti, Tosini, arXiv:1603.06442





Weyl 3d



D'Ariano, Mosco, Perinotti, Tosini, arXiv:1603.06442

Dirac 3d

```
mass:       0.002
sigma:      32
x0:         [140,140,140]
k0:         [0.05,0.05,0.05]
spinor:     ["Exp[I k0.#]",0,0,0]
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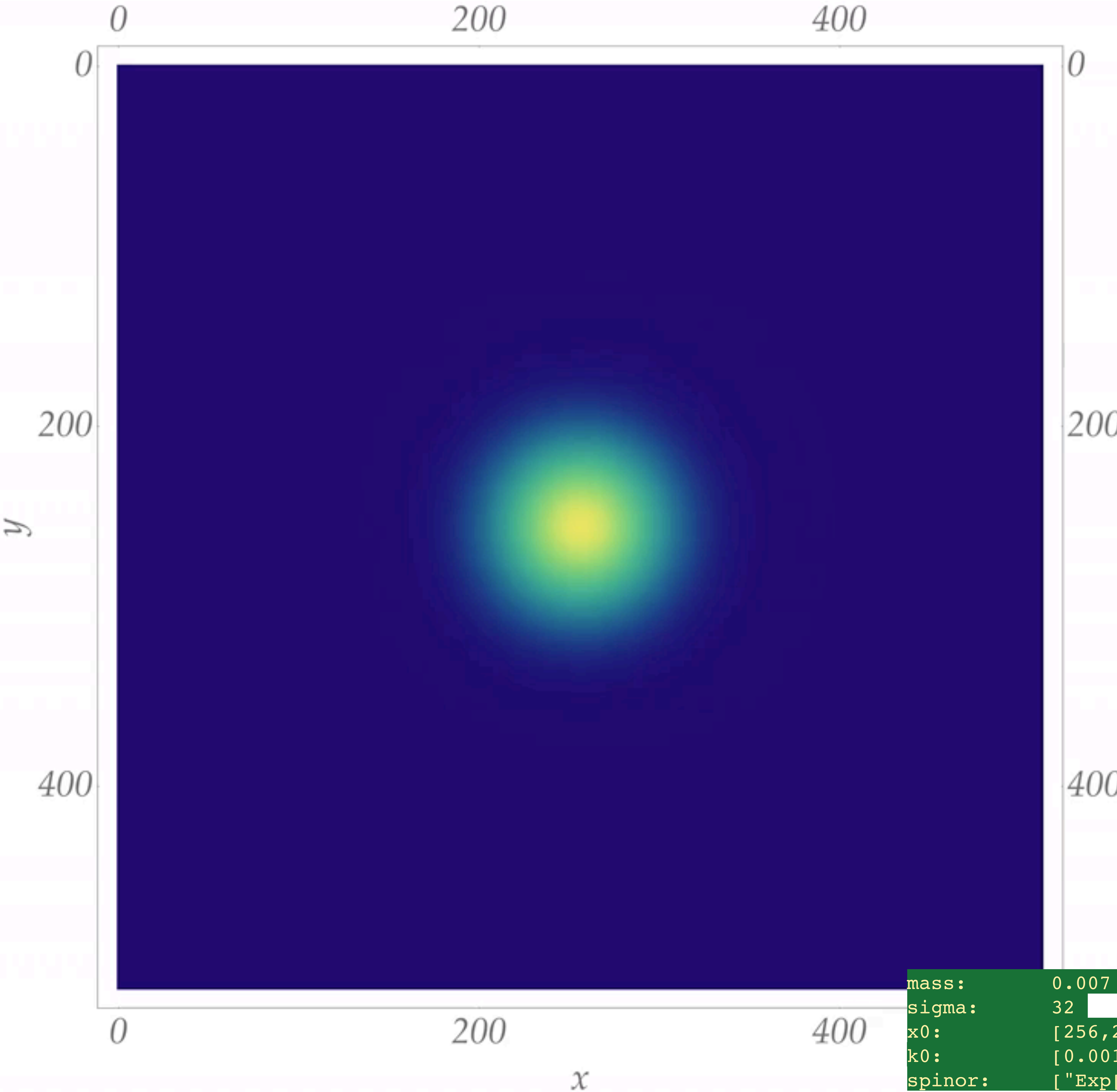
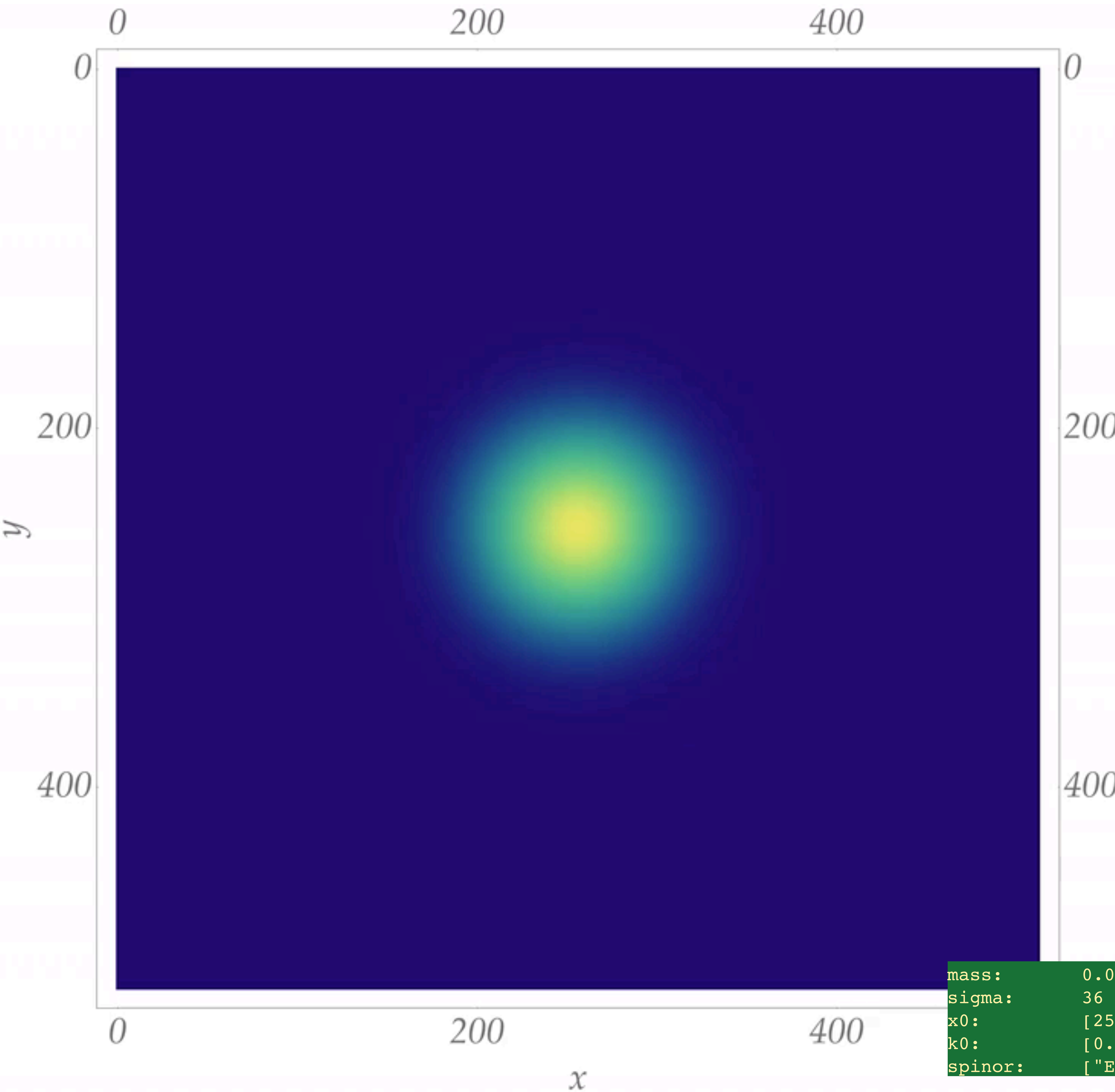


D'Ariano, Mosco, Perinotti, Tosini, arXiv:1603.06442

Dirac 3d

```
mass:        0.007
sigma:       32
x0:          [256,256,256]
k0:          [0.001,0.001,0.001]
spinor:      ["Exp[I k0.#]",0,0,"Exp[I k0.#]"]
```



D'Ariano, Mosco, Perinotti, Tosini, arXiv:1603.06442

Dirac 3d

```
mass:       0.008
sigma:      36
x0:         [256,256,256]
k0:         [0.05,0.05,0.05]
spinor:     ["Exp[I k0.#]",0,0,"Exp[I k0.#]"]
```

Dirac emerging from the QCA

D'Ariano, Perinotti,
PRA **90** 062106 (2014)

fidelity with Dirac for a narrowband packets in the relativistic limit $k \simeq m \ll 1$

$$F = |\langle \exp\left[-iN\boldsymbol{\Delta}(\mathbf{k})\right]\rangle|$$

$$\begin{aligned}\boldsymbol{\Delta}(\mathbf{k}) &:= (m^2 + \tfrac{k^2}{3})^{\frac{1}{2}} - \omega^E(\mathbf{k}) \\ &= \frac{\sqrt{3}k_x k_y k_z}{(m^2 + \frac{k^2}{3})^{\frac{1}{2}}} - \frac{3(k_x k_y k_z)^2}{(m^2 + \frac{k^2}{3})^{\frac{3}{2}}} + \tfrac{1}{24}(m^2 + \tfrac{k^2}{3})^{\frac{3}{2}} + \mathcal{O}(k^4 + N^{-1}k^2)\end{aligned}$$

relativistic proton: $N \simeq m^{-3} = 2.2 * 10^{57} \;\Rightarrow\; t = 1.2 * 10^{14}\mathrm{s} = 3.7 * 10^6$ y

UHECRs: $k = 10^{-8} \gg m \;\Rightarrow N \simeq k^{-2} = 10^{16} \;\Rightarrow\; 5 * 10^{-28}$ s

G. M. D'Ariano, N. Mosco, P. Perinotti, A. Tosini,
PLA **378** 3165 (2014); EPL **109** 40012 (2015)

Analytical solution of Dirac (d=1) and Weyl (d=1,2,3)

The analytical solution of the Dirac automaton can also be expressed in terms of Jacobi polynomials $P_k^{(\zeta,\rho)}$ performing the sum over f in Eq. (16) which finally gives

$$\psi(x,t) = \sum_y \sum_{a,b\in\{0,1\}} \gamma_{a,b} P_k^{(1,-t)}\left(1+2\left(\frac{m}{n}\right)^2\right) A_{ab}\psi(y,0),$$
$$k = \mu_+ - \frac{a\oplus b+1}{2},$$
$$\gamma_{a,b} = -(\mathrm{i}^{a\oplus b}) n^t \left(\frac{m}{n}\right)^{2+a\oplus b} \frac{k!\left(\mu_{(-)^{ab}} + \frac{\overline{a\oplus b}}{2}\right)}{(2)_k}, \tag{18}$$

where $\gamma_{00} = \gamma_{11} = 0$ ($\gamma_{10} = \gamma_{01} = 0$) for $t+x-y$ odd (even) and $(x)_k = x(x+1)\cdots(x+k-1)$.

Dispersive Schrödinger equation

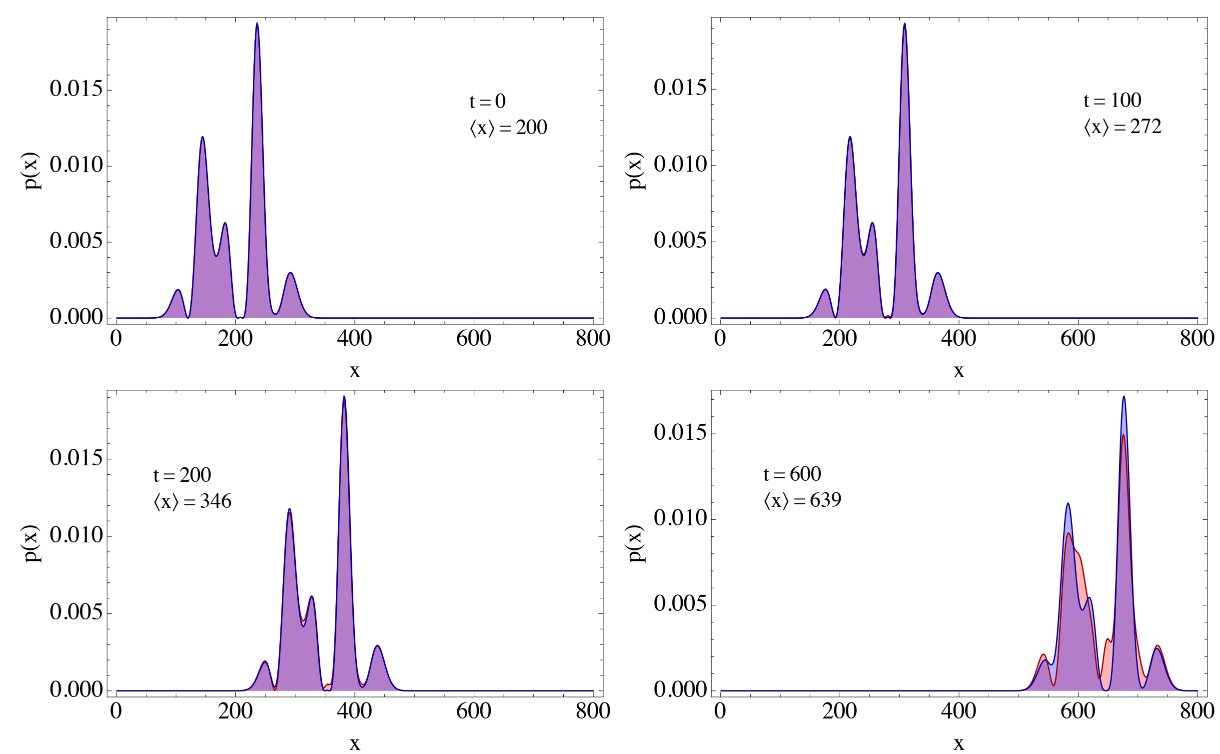
$$i\partial_t e^{-i\mathbf{k}_0\cdot\mathbf{x}+i\omega_0 t}\psi(\mathbf{k},t) = s[\omega(\mathbf{k})-\omega_0]e^{-i\mathbf{k}_0\cdot\mathbf{x}+i\omega_0 t}\psi(\mathbf{k},t)$$

$$i\partial_t\tilde{\psi}(\mathbf{k},t) = s[\omega(\mathbf{k})-\omega_0]\tilde{\psi}(\mathbf{k},t)$$

$$i\partial_t\tilde{\psi}(\mathbf{x},t) = s[\mathbf{v}\cdot\boldsymbol{\nabla}+\frac{1}{2}\mathbf{D}\cdot\boldsymbol{\nabla}\boldsymbol{\nabla}]\tilde{\psi}(\mathbf{x},t)$$

$$\mathbf{v} = (\boldsymbol{\nabla}_{\mathbf{k}}\omega)(\mathbf{k}_0)$$

$$\mathbf{D} = (\boldsymbol{\nabla}_{\mathbf{k}}\boldsymbol{\nabla}_{\mathbf{k}}\omega)(\mathbf{k}_0)$$



D'Ariano, Perinotti, PRA **90** 062106 (2014)

Bisio, D'Ariano, Perinotti, unpublished

Special Relativity recovered … and more

Relativity principle: invariance of the physical law under change of inertial reference frame
→ invariance of eigenvalue equation under change of representation.

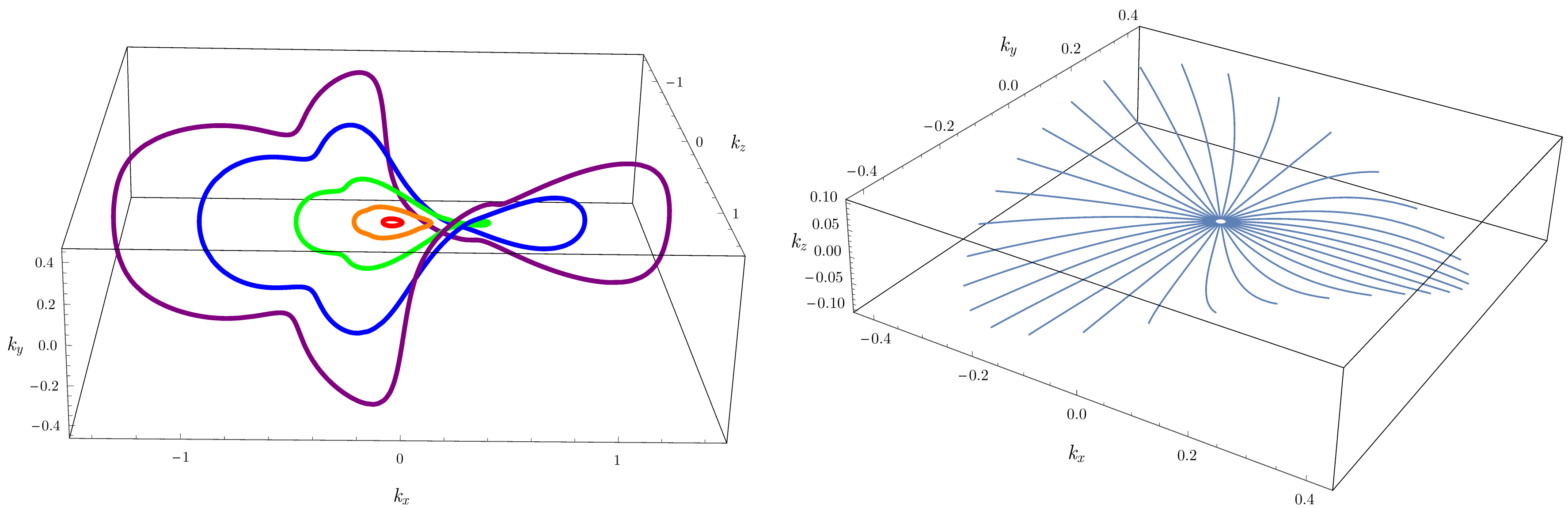
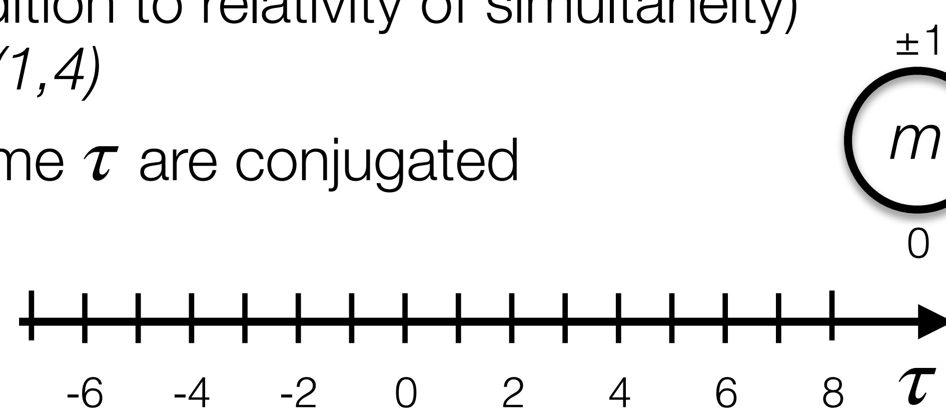


FIG. 2: The distortion effects of the Lorentz group for the discrete Planck-scale theory represented by the quantum walk in Eq. (6). Left figure: the orbit of the wavevectors $\mathbf{k} = (k_x, 0, 0)$, with $k_x \in \{.05, .2, .5, 1, 1.7\}$ under the rotation around the z axis. Right figure: the orbit of wavevectors with $|\mathbf{k}| = 0.01$ for various directions in the (k_x, k_y) plane under the boosts with $\boldsymbol{\beta}$ parallel to $\mathbf{k}$ and $|\boldsymbol{\beta}| \in [0, \tanh 4]$.

- Lorentz transformations are perfectly recovered for *k≪1.*
- For *k~1*:
  - *Double Special Relativity* (Camelia-Smolin).
  - *Relative locality* (in addition to relativity of simultaneity)
  - For *m≠0 De Sitter SO(1,4)*
  - mass *m* and proper-time τ are conjugated



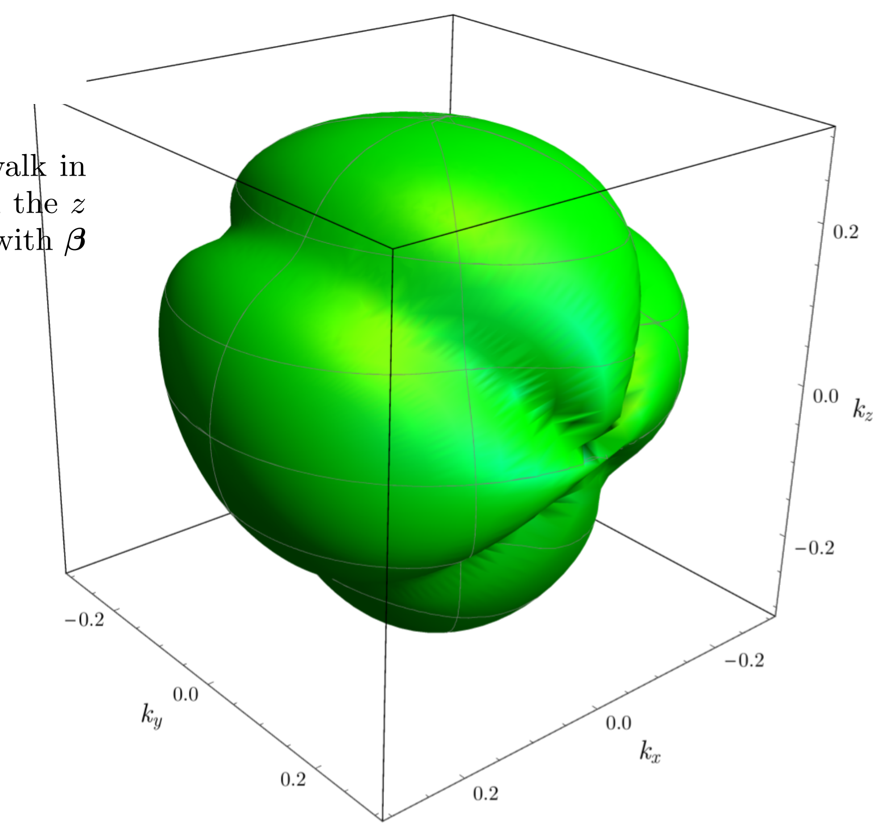


FIG. 3: The green surface represents the orbit of the wavevector $\mathbf{k} = (0.3, 0, 0)$ under the full rotation group $SO(3)$.

Things we would like to know

Things we would like to know

A QWCG for G qi-embeddable in H^2 would provide a Weyl/Dirac free QFT on a curved space, without using quantization rules. A decomposition into irreps. of the right-regular rep. for a finitely presented group G qi-embeddable in H^2 would be needed.

1. Is there a result of *qi-rigidity* (similar to that for R^d) for H^2? What about a generic Riemannian manifold M with dimension $d=1,2,3$?

Is the free group F^2 qi to H^2? What about the converse, namely: if G is qi to H^2 then G is virtually free?

If the previous statement is true, then the right-reg. representation of F^2 would provide the right-reg. representation of the virtually-free group through induced representation (*"renormalization"*).

Things we would like to know

2. How the condition of <u>symmetric</u> Cayley graph restricts the structure of its G? [*]

3. Given G, S_+, and $s>0$, find all the unitary nonequivalent sets of matrices $\{A_h\}_{h\in S}$ that provide a nontrivial WQ $Q=(G, S_+, s, \{A_h\}_{h\in S})$.

The unitarity equations for the transfer matrices $\{A_h\}_{h\in S}$ depend only on $|S_+|$ and on the 4-cycles.

4. Do groups sharing the same 4-cycles have something in common?

5. Does the property of being presentable with relators $|r|\leq 4$ correspond to some group property?

[*] A directed colored graph is symmetric if its automorphism group acts transitively upon ordered pairs of adjacent vertices.

Things we would like to know

6. Given a group *G* with Cayley graph qi to a smooth Riemanian manifold *M* with a nonzero curvature, can a “tangent group” be defined (similarly to what we do for tangent space to *M*) as a sort of “local Abelianization” of *G*? The QWCG on *G* should correspond to a Schrödinger equation with a Laplace-Beltrami diffusion on *M.*

7. What happens for negative curvature (exponential growth of *G*?)

8. What is the equivalent of Fourier-transform decomposition into irreps. for finitely presented hyperbolic *G?* What is the notion of wave-vector *k?* What does it mean $k \ll 1$ (relativistic regime)?

9. The universal covering of an arbitrary graph is a Cayley graph. Given a QW on a graph, can a QW be induced on his universal covering (and viceversa)?

This is more or less what I wanted to say

Thank you for your attention