Relativity principle without space-time

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Is quantum theory exact?
The endeavor for the theory beyond standard quantum mechanics.
Second Edition FQT2015

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Program

Derive the whole Physics from principles

Physics as an axiomatic theory

with thorough physical interpretation

Principles for Quantum Theory

Selected for a Viewpoint in *Physics*

PHYSICAL REVIEW A **84**, 012311 (2011)

Informational derivation of quantum theory

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We derive quantum theory from purely informational principles. Five elementary axioms—causality, perfect distinguishability, ideal compression, local distinguishability, and pure conditioning—define a broad class of theories of information processing that can be regarded as standard. One postulate—purification—singles out quantum theory within this class.

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Principles for Quantum Theory

P1. Causality

P2. Local discriminability

P3. Purification

P4. Atomicity of composition

P5. Perfect distinguishability

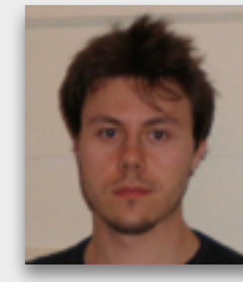
P6. Lossless Compressibility

Book from CUP soon!

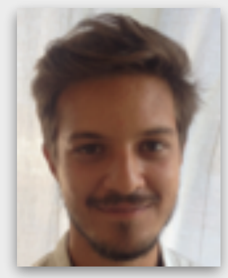
Principles for Mechanics

Paolo Perinotti

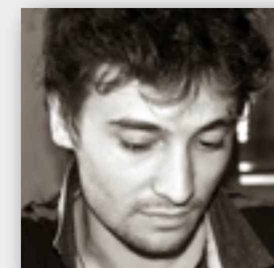
Alessandro Bisio



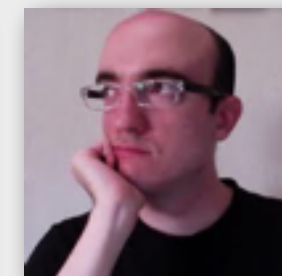
Alessandro Tosini



Marco Erba



Franco Manessi



Nicola Mosco

- *Mechanics (QFT) derived in terms of countably many quantum systems in interaction*

add principles

Min algorithmic complexity principle

- homogeneity
- locality
- reversibility

Principles for Quantum Field Theory

- *QFT derived in terms of countably many quantum systems in interaction*

add principles

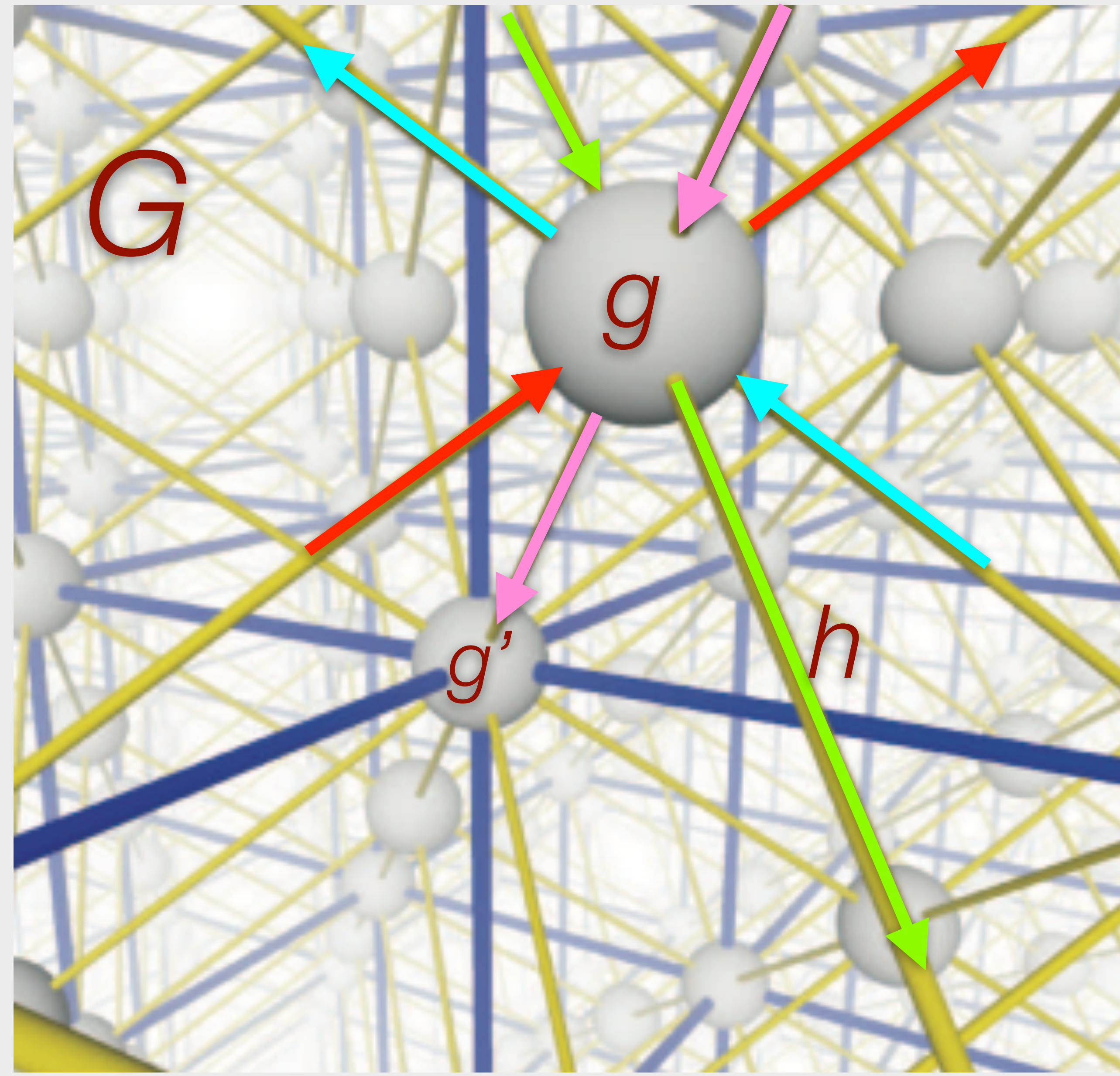
Min algorithmic complexity principle

- homogeneity
- locality
- reversibility

Quantum Cellular Automata on the Cayley graph of a group *G*

- linearity
- isotropy
- minimal-dimension
- Cayley qi-embedded in R^d

Restrictions



$$G=\langle h_1,h_2,\dots|r_1,r_2,\dots\rangle=:\langle S_+|R\rangle$$

Principles for Quantum Field Theory

- *QFT derived in terms of countably many quantum systems in interaction*

add principles

Min algorithmic complexity principle

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- reversibility

Quantum Cellular Automata on the Cayley graph of a group *G*

- linearity
- isotropy
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Restrictions

G virtually Abelian

(geometric group theory)

Linearity (free QFT)

Quantum Cellular Automaton ⇒ *Quantum Walk*

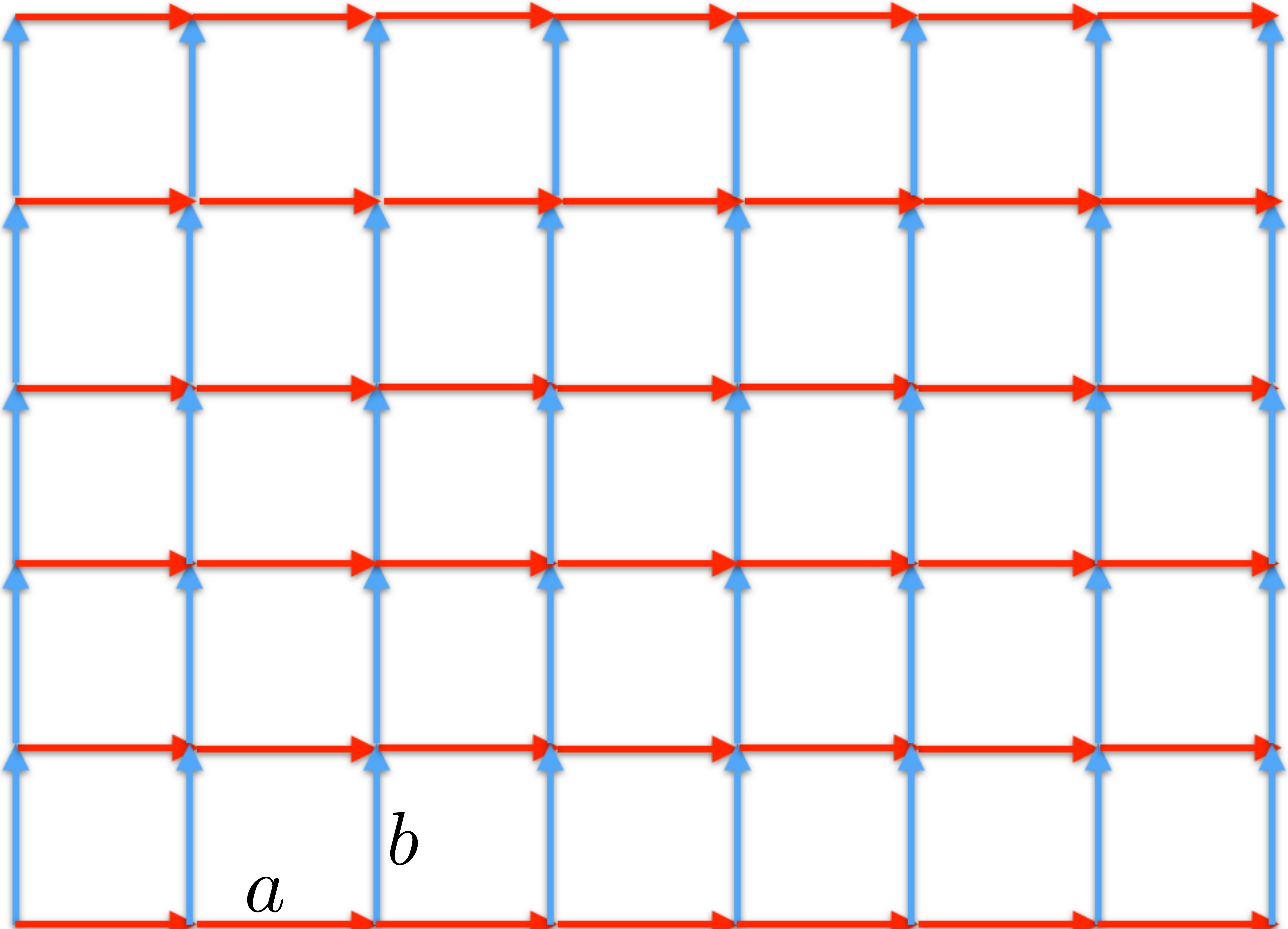
$$U\psi U^\dagger = A\psi$$

von Neumann algebra ⇒ Fock space

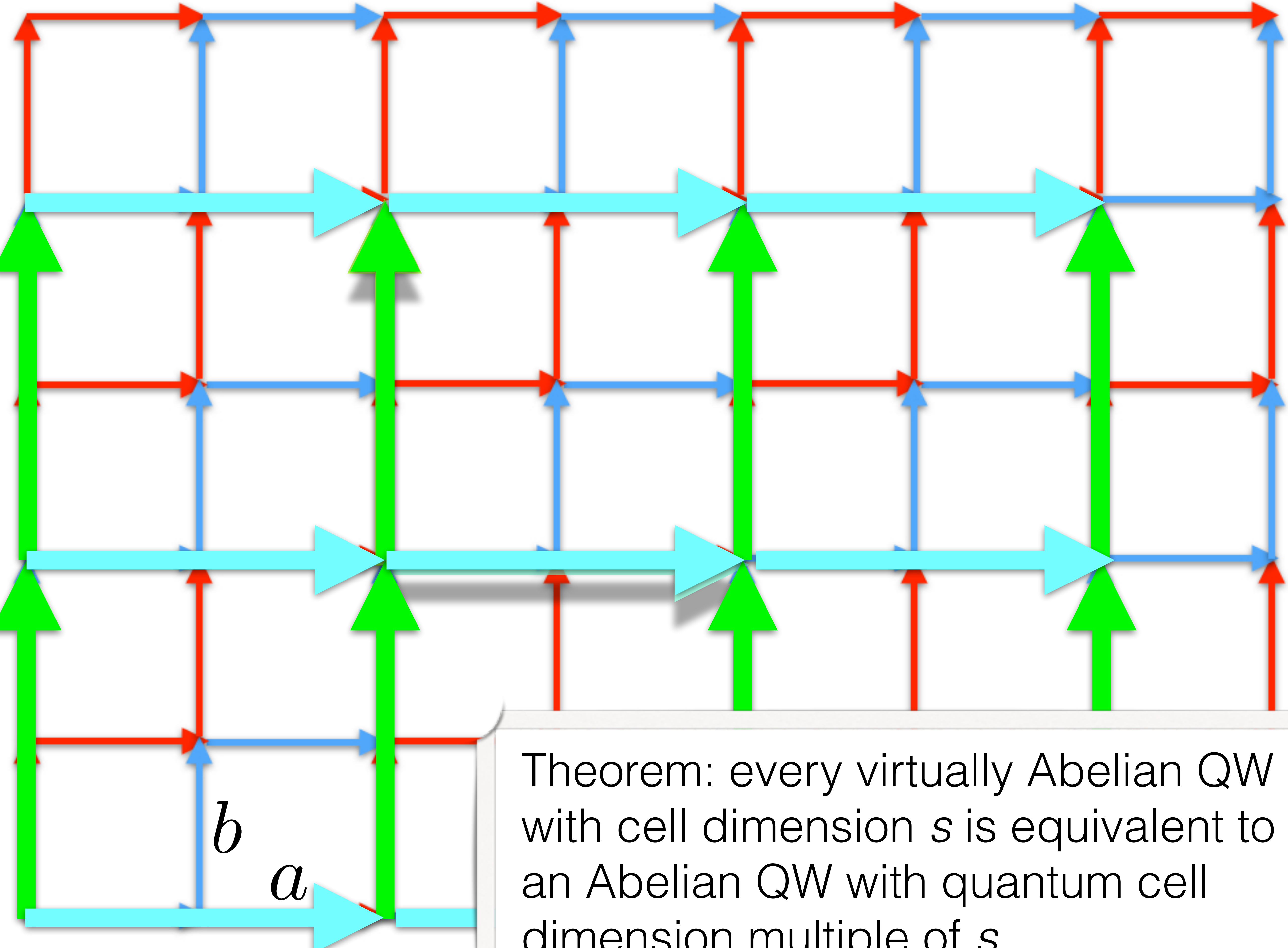
Isotropy

- There exists a group L of permutations of S_+, transitive over S_+ that leaves the Cayley graph invariant
- a nontrivial unitary s-dimensional (projective) representation $\{L_l\}$ of L such that:

$$\boxed{A = \sum_{h\in S} T_h \otimes A_h = \sum_{h\in S} T_{lh} \otimes L_l A_h L_l^\dagger}$$



$$G = \langle a, b | aba^{-1}b^{-1} \rangle \equiv Z \times Z$$



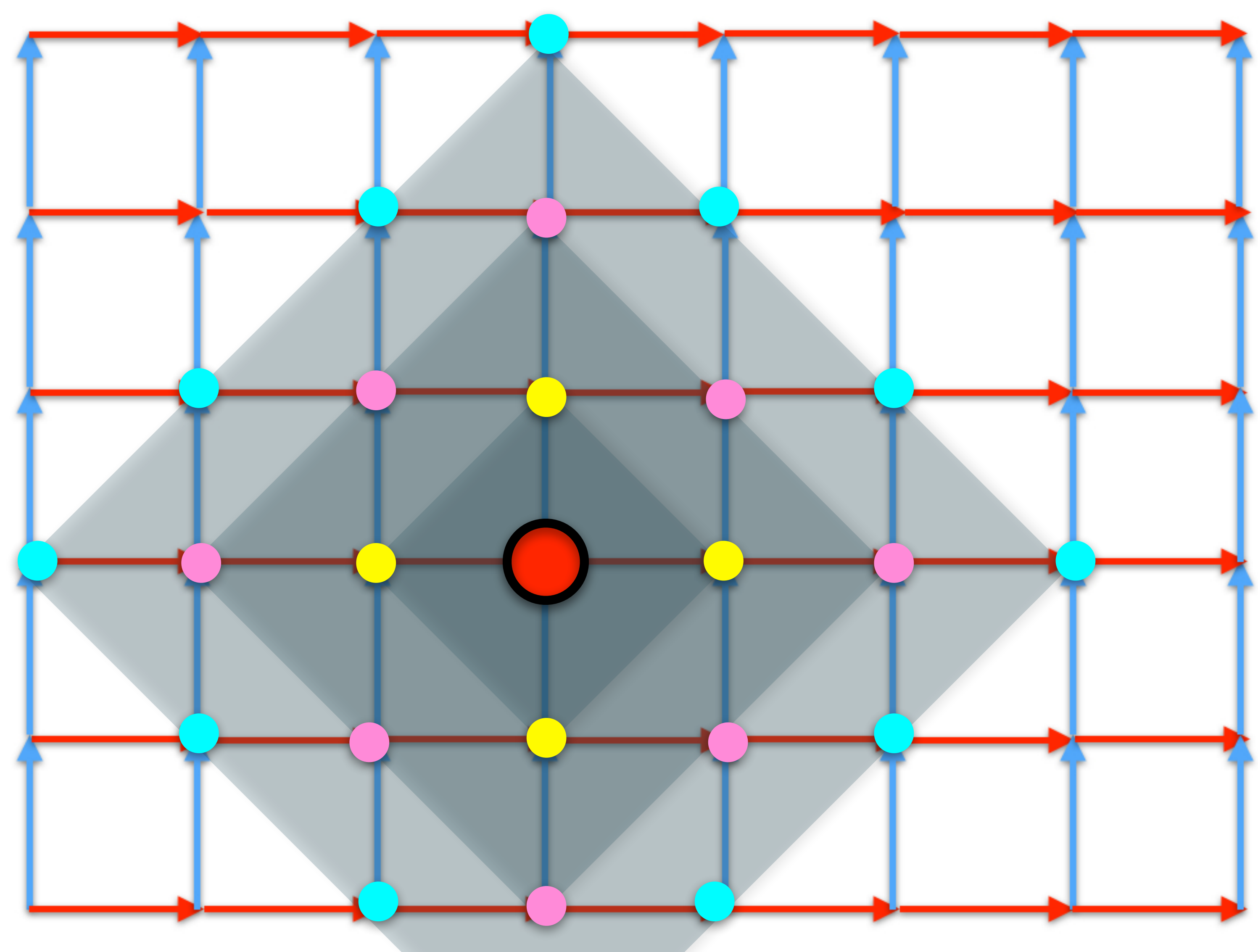
Theorem: every virtually Abelian QW with cell dimension *s* is equivalent to an Abelian QW with quantum cell dimension multiple of *s*.

Quantum walk on Cayley graph

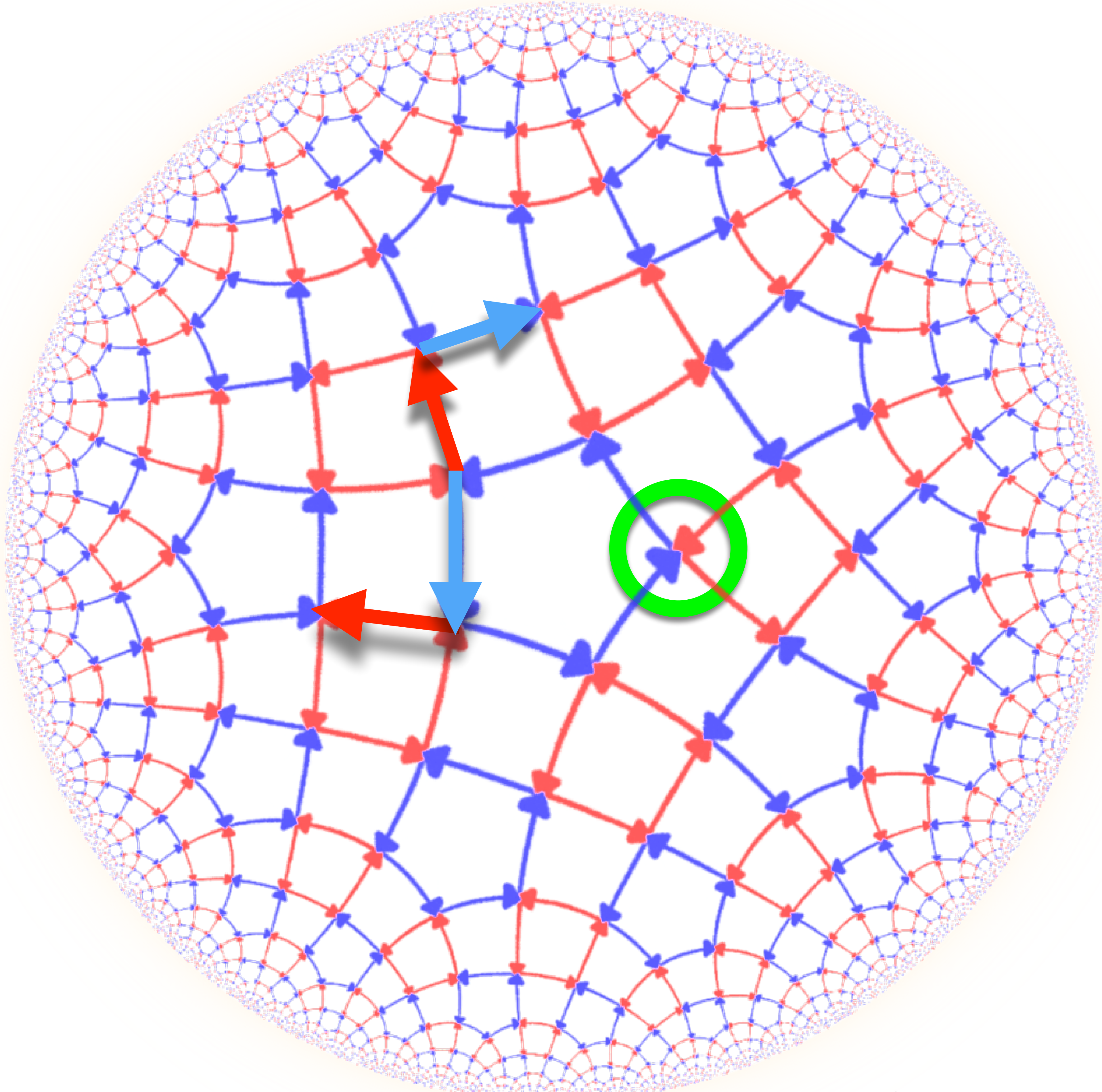
Theorem: A group is quasi-isometrically embeddable in R^d iff it is virtually Abelian

Virtually Abelian groups have polynomial growth (Gromov)

points $\sim r^d$

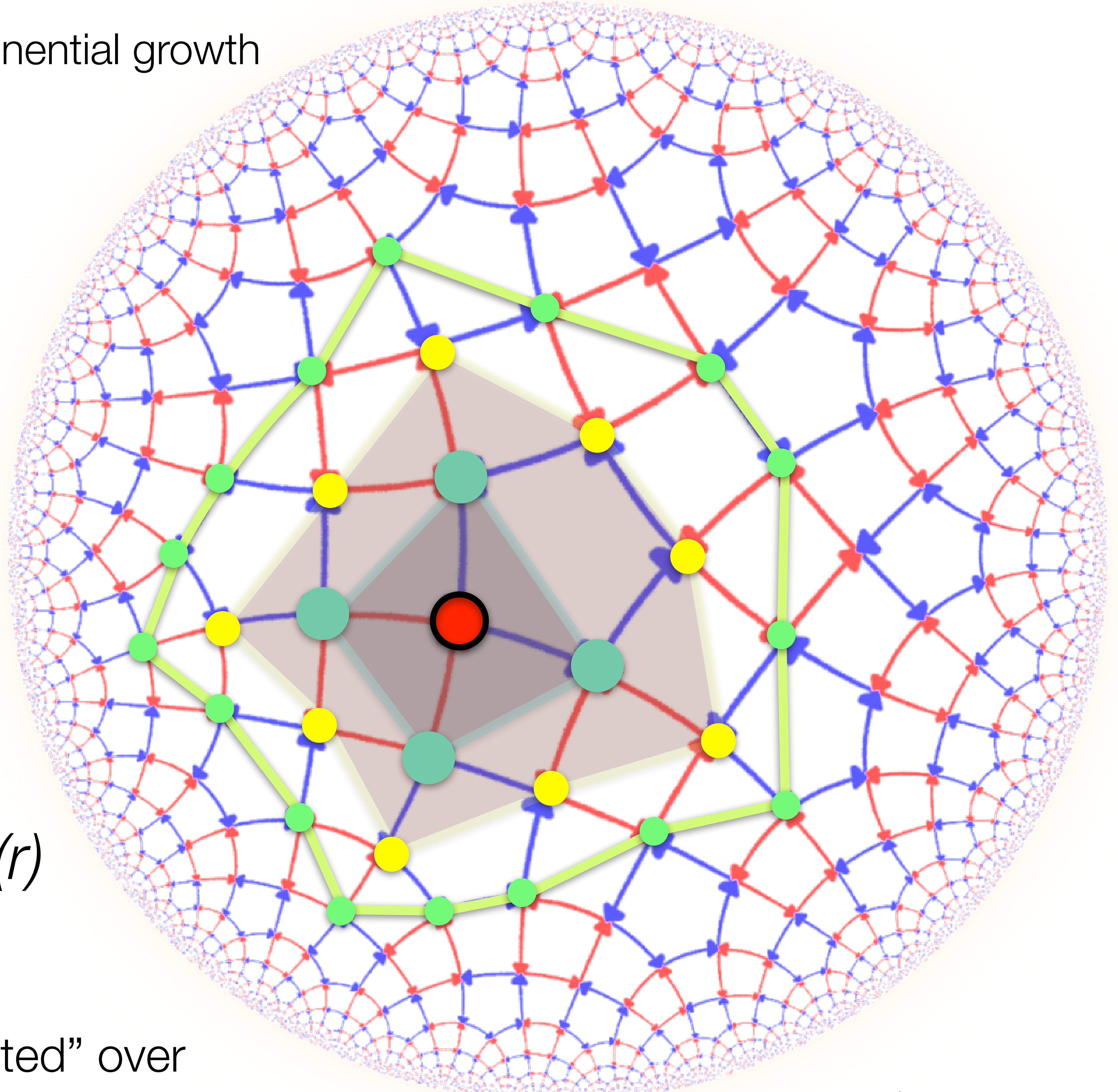


- G hyperbolic



$$G = \langle a, b | a^5, b^4, (ab)^2 \rangle$$

- *G* hyperbolic → exponential growth



\# points ~ *exp(r)*

information “transmitted” over
the graph decreases as *exp(-r)*

$$G = \langle a, b | a^5, b^4, (ab)^2 \rangle$$

Informationalism: Principles for QFT

- *QFT derived in terms of countably many quantum systems in interaction*

add principles

Min algorithmic complexity principle

- homogeneity
- locality
- reversibility

Quantum Cellular Automata on the Cayley graph of a group *G*

- linearity
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Restrictions

G virtually Abelian

- *Relativistic regime (k«1):* free QFT (Weyl, Dirac, and Maxwell)
- *Ultra-relativistic regime (k~1)* [Planck scale]: nonlinear Lorentz

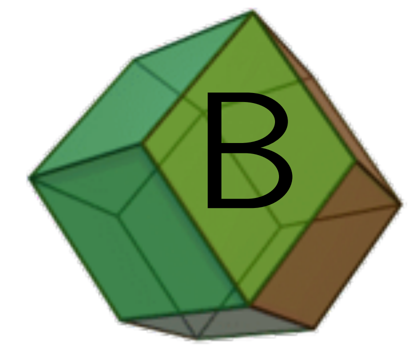
- QFT derived:
  - without assuming Special Relativity
  - without assuming mechanics (quantum *ab-initio*)
- QCA is a *discrete* theory

Motivations to keep it discrete:

1. Discrete contains continuum as special regime
2. Testing mechanisms in quantum simulations
3. Falsifiable discrete-scale hypothesis
4. Natural scenario for holographic principle
5. Solves all issues in QFT originating from continuum:
   i) uv divergencies
   ii) localization issue
   iii) Path-integral
6. Fully-fledged theory to evaluate cutoffs

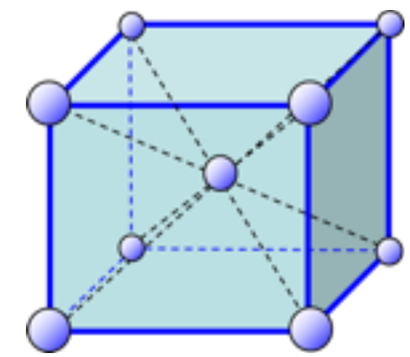
The Weyl QCA

☞ Minimal dimension for nontrivial unitary Abelian QW is *s=2*



Qi-embeddability in R^3

- Uni+Iso ⇒ the only possible *Cayley* is the BCC!!
- Iso ⇒ Fermionic ψ (*d=3*)



Unitary operator: $$A = \int_B^{\oplus} d\mathbf{k}\, A_{\mathbf{k}}$$

Two QWs connected by P

$$\boxed{\begin{aligned} A_{\mathbf{k}}^{\pm} = &- i\sigma_x(s_x c_y c_z \pm c_x s_y s_z) \\ &\mp i\sigma_y(c_x s_y c_z \mp s_x c_y s_z) \\ &- i\sigma_z(c_x c_y s_z \pm s_x s_y c_z) \\ &+ I(c_x c_y c_z \mp s_x s_y s_z) \end{aligned}}$$

$$s_\alpha = \sin\frac{k_\alpha}{\sqrt{3}}$$

$$c_\alpha = \cos\frac{k_\alpha}{\sqrt{3}}$$

D'Ariano, Perinotti, PRA **90** 062106 (2014)

The Weyl QCA

$$i\partial_t\psi(t) \simeq \tfrac{i}{2}[\psi(t+1)-\psi(t-1)] = \tfrac{i}{2}(A-A^\dagger)\psi(t)$$

$$\begin{aligned}\tfrac{i}{2}(A^{\pm}_{\mathbf{k}}-A^{\pm\dagger}_{\mathbf{k}}) = &+\sigma_x(s_xc_yc_z \pm c_xs_ys_z)\\ &\pm\sigma_y(c_xs_yc_z \mp s_xc_ys_z)\\ &+\sigma_z(c_xc_ys_z \pm s_xs_yc_z)\end{aligned}$$

"Hamiltonian"

$k \ll 1$

$$\boxed{i\partial_t\psi = \tfrac{1}{\sqrt{3}}\boldsymbol{\sigma}^{\pm}\cdot\mathbf{k}\,\psi}$$ ☜ Weyl equation!

$$\boldsymbol{\sigma}^{\pm} := (\sigma_x, \pm\sigma_y, \sigma_z)$$

Two QCAs connected by P

$$\boxed{\begin{aligned}A^{\pm}_{\mathbf{k}} = &-i\sigma_x(s_xc_yc_z \pm c_xs_ys_z)\\ &\mp i\sigma_y(c_xs_yc_z \mp s_xc_ys_z)\\ &-i\sigma_z(c_xc_ys_z \pm s_xs_yc_z)\\ &+I(c_xc_yc_z \mp s_xs_ys_z)\end{aligned}}$$

$$s_\alpha = \sin\frac{k_\alpha}{\sqrt{3}}$$

$$c_\alpha = \cos\frac{k_\alpha}{\sqrt{3}}$$

D'Ariano, Perinotti, PRA **90** 062106 (2014)

Dirac QCA $\oplus$

Local coupling: $A_{\mathbf{k}}$ coupled with its inverse with off-diagonal identity block matrix

$$\boxed{\begin{aligned} E_{\mathbf{k}}^{\pm} &= \begin{pmatrix} nA_{\mathbf{k}}^{\pm} & imI \\ imI & nA_{\mathbf{k}}^{\pm\dagger} \end{pmatrix} \\ n^2 + m^2 &= 1 \end{aligned}}$$

$E_{\mathbf{k}}^{\pm}$ CPT-connected!

$$\omega_{\pm}^{E}(\mathbf{k}) = \cos^{-1}[n(c_x c_y c_z \mp s_x s_y s_z)]$$

Dirac in relativistic limit $k \ll 1$

m≤1: mass

n^{-1}: refraction index

Bisio, D'Ariano, Perinotti, arXiv:1407.6928

Maxwell QCA $\otimes$

$$\boxed{\begin{aligned} M_{\mathbf{k}} &= A_{\mathbf{k}} \otimes A_{\mathbf{k}}^{*} \\ F^{\mu}(\mathbf{k}) &= \int \frac{\mathrm{d}\,\mathbf{q}}{2\pi} f(\mathbf{q}) \tilde{\psi}(\tfrac{\mathbf{k}}{2} - \mathbf{q}) \sigma^{\mu} \varphi(\tfrac{\mathbf{k}}{2} + \mathbf{q}) \end{aligned}}$$

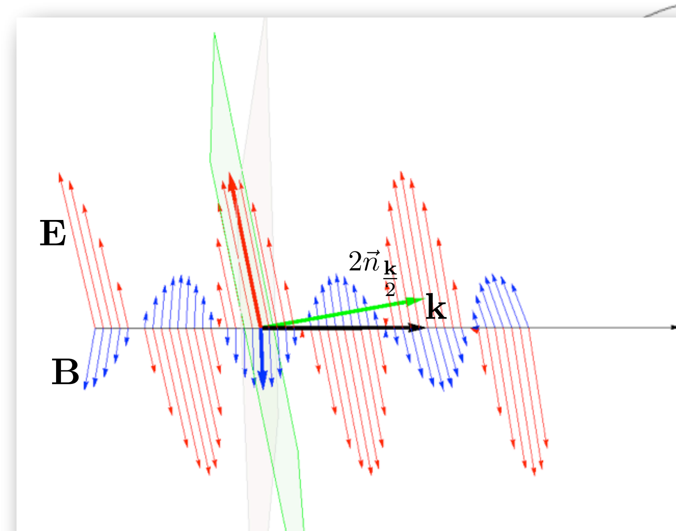
Maxwell in relativistic limit $k \ll 1$

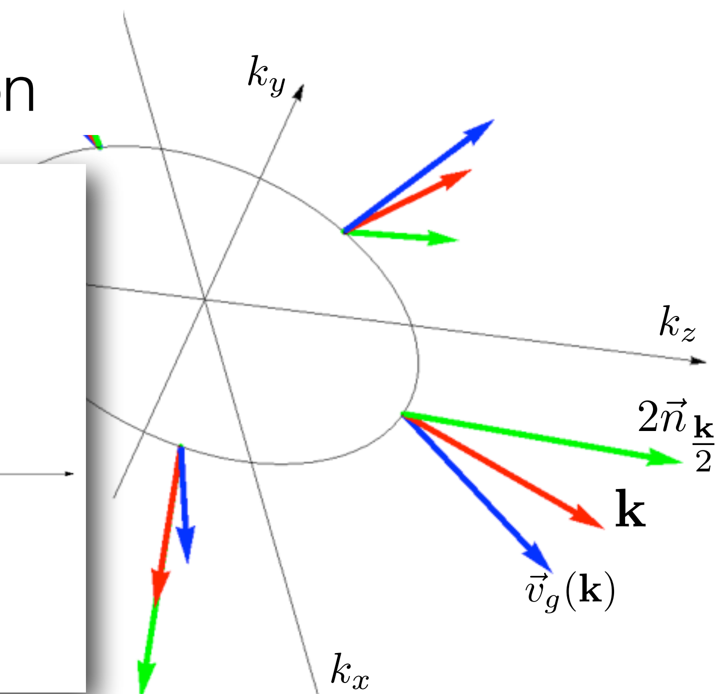
Boson: emergent from entangled Fermions

(De Broglie neutrino-theory of photon)

$$c^{\mp}(\mathbf{k}) = c\left(1 \pm \frac{k_x k_y k_z}{|\mathbf{k}|^2}\right)$$

1. Vacuum birefringence
2. Vacuum dispersion
3. Fermionic saturation





The LTM standards of the theory

Dimensionless variables

$$x = \frac{x_{[m]}}{\mathfrak{a}} \in \mathbb{Z}, \quad t = \frac{t_{[sec]}}{\mathfrak{t}} \in \mathbb{N}, \quad m = \frac{m_{[kg]}}{\mathfrak{m}} \in [0,1]$$

Relativistic limit: $\Rightarrow$ $\boldsymbol{c} = \mathfrak{a}/\mathfrak{t} \qquad \boldsymbol{\hbar} = \mathfrak{m}\mathfrak{a}\boldsymbol{c}$

Measure $\mathfrak{m}$ from mass-refraction-index

$$\Rightarrow \quad n(m_{[kg]}) = \sqrt{1 - \left(\frac{m_{[kg]}}{\mathfrak{m}}\right)}$$

Measure $\mathfrak{a}$ from light-refraction-index

$$\Rightarrow \quad c^{\mp}(k) = c\left(1 \pm \frac{k}{\sqrt{3}k_{max}}\right)$$

The relativity principle

Symmetries and Relativity Principle

Looking for changes of reference-frames that leaves the dynamics invariant

Change of reference-frame = special change of representation

VA QWs

$$A = \int_B^{\oplus} d\mathbf{k}\, A_{\mathbf{k}}$$

$$\mathbf{n}(\mathbf{k}) \cdot \mathbf{T} := \frac{i}{2}(A_{\mathbf{k}} - A_{\mathbf{k}}^{\dagger})$$ "Hamiltonian"

$\mathbf{n}(\mathbf{k})$ analytic in $\mathbf{k}$

$(I, \mathbf{T}) = (T^{\mu})$ Hermitian basis for $\mathrm{Lin}(\mathbb{C}^s)$

Dynamics: eigenvalue equation

$$A_{\mathbf{k}}\psi(\mathbf{k},\omega) = e^{i\omega}\psi(\mathbf{k},\omega)$$

$$(\sin\omega I - \mathbf{n}(\mathbf{k}) \cdot \mathbf{T})\psi(\mathbf{k},\omega) = 0$$

For each value of $\mathbf{k}$ there are at most s eigenvalues $\{\omega_l(\mathbf{k})\}$

$\mathbf{n}(\mathbf{k})$ analytic in $\mathbf{k}$ + finite-dim irreps.

$\omega_l(\mathbf{k})$ continuous

dispersion relations branches

Symmetries and Relativity Principle

Change of reference-frame: $(\omega, \mathbf{k}) \to (\omega', \mathbf{k}') = \mathcal{L}_\beta(\omega, \mathbf{k})$

$\mathcal{L}_\beta$ invertible (generally non continuous) over $[-\pi, \pi] \times$ B

$\{\mathcal{L}_\beta\}_{\beta \in \mathbb{G}}$ $\mathbb{G}$ group (including space-inversion, charge conjugation,...)

Symmetry of the dynamics:

there exists a pair of invertible matrices Γ_β and $\tilde{\Gamma}_\beta$ such that the following identity holds:

$$(\sin\omega I - \mathbf{n}(\mathbf{k}) \cdot \mathbf{T}) = \tilde{\Gamma}_\beta^{-1}(\sin\omega' I - \mathbf{n}(\mathbf{k}') \cdot \mathbf{T})\Gamma_\beta$$

Γ_β and $\tilde{\Gamma}_\beta$ continuous functions of $(\omega, \mathbf{k})$

$\tilde{\Gamma}_\beta, \Gamma_\beta$ can also contain LUs, gauge-transforms, ...,

$\mathbb{G}_0$ (id-component of $\mathbb{G}$) preserves the branches

change of reference-frame = $\mathbf{k} \to \mathbf{k}'(\mathbf{k})$

change of reference-frame = reshuffling $\mathbf{k} \to \mathbf{k}'(\mathbf{k})$ of irreps. holds for the whole class of VA QW

$\mathbf{k} \to \mathbf{k}'(\mathbf{k})$

$\mathcal{L}_\beta(\omega, \mathbf{k}) = (\omega(\mathbf{k}'), \mathbf{k}'(\mathbf{k}))$

$\mathbb{G}_0, \mathbb{G}$ depend on the QW!

Bisio, D'Ariano, Perinotti, arXiv:1503.01017

Relativity Principle for Weyl QW

$$p^{(f)} := f(\omega, \mathbf{k})(\sin\omega, \mathbf{n}(\mathbf{k}))$$

$p_\mu p^\mu = 0$ on $\mathrm{Disp}(A)$

$p^{(f)}_\mu \sigma^\mu \psi(\mathbf{k}, \omega) = 0$ "4-momentum"

Non-linear Lorentz group

$$\mathcal{L}^{(f)}_\beta := \mathcal{D}^{(f)-1} L_\beta \mathcal{D}^{(f)}$$

$$\mathcal{D}^{(f)} : (\omega, \mathbf{k}) \mapsto p^{(f)}(\omega, \mathbf{k})$$

acting on $[-\pi, \pi] \times$ B

$\mathrm{Disp}(A)$ invariant $\Rightarrow$ L_β Lorentz

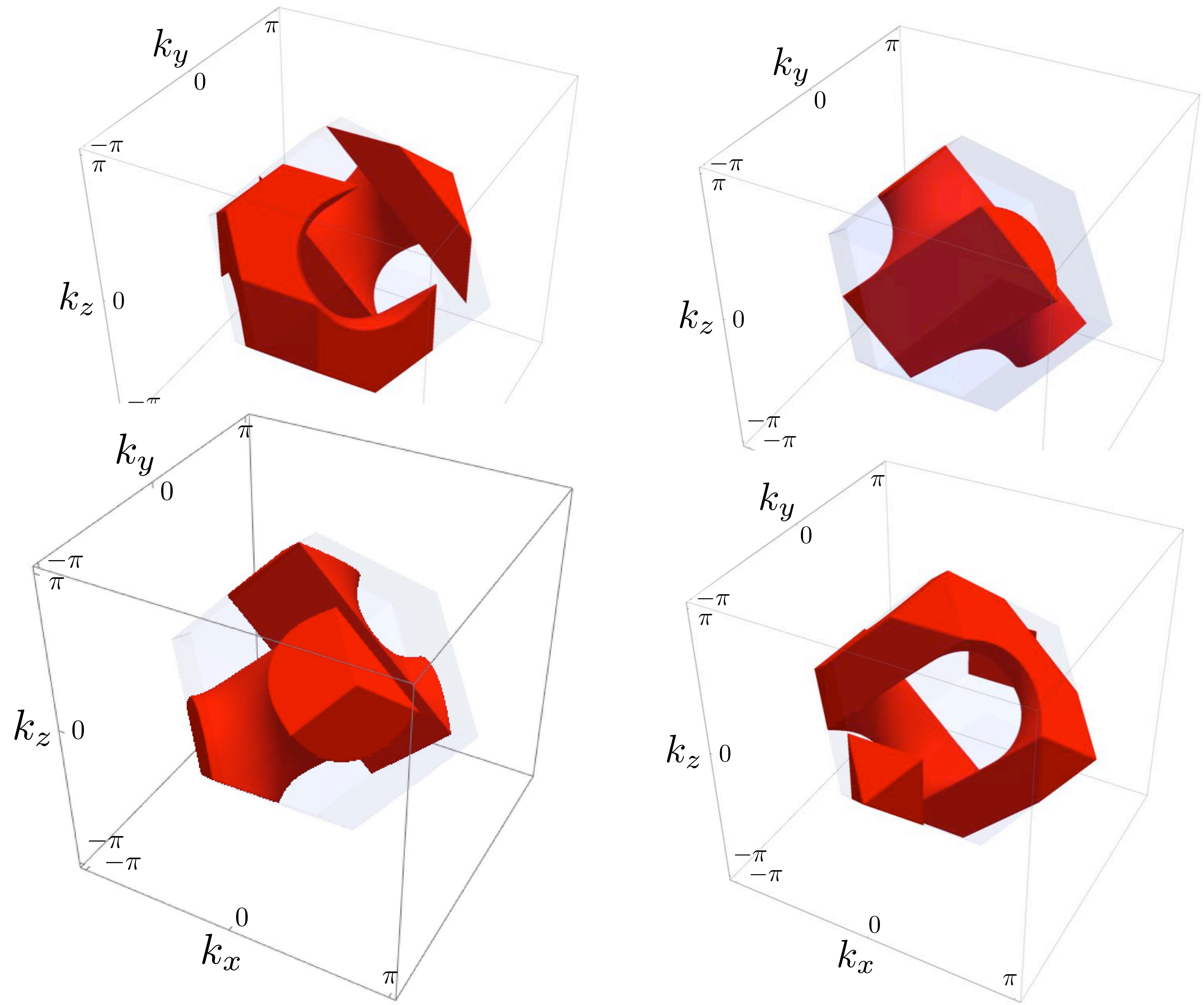
Relativistic covariance of dynamics

$$(\sin\omega I - \mathbf{n}(\mathbf{k}) \cdot \boldsymbol{\sigma}) = \tilde{\Lambda}^\dagger_\beta (\sin\omega' I - \mathbf{n}(\mathbf{k}') \cdot \boldsymbol{\sigma}) \Lambda_\beta$$

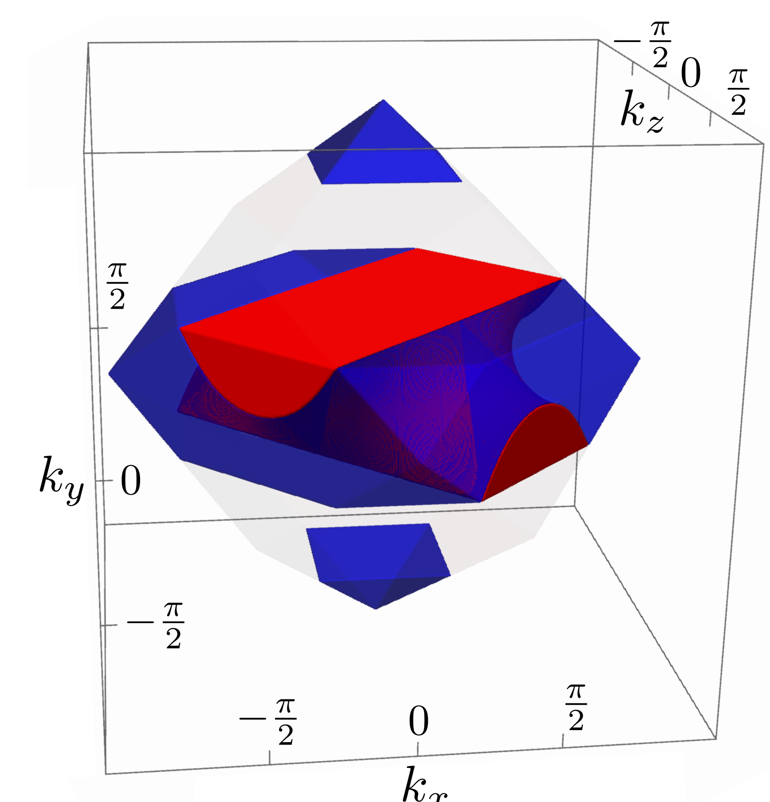
$\Lambda_\beta \in \mathrm{SL}_2(\mathbb{C})$ independent of (k_μ)

Relativity Principle for Weyl QW

Includes the group of “translations” of the Cayley graph: $\mathbb{G}_0$ is the Poincaré group



The Brillouin zone separates into ***four invariant regions*** diffeomorphic to balls, corresponding to four different ***particles***.



Relativity Principle for Dirac QW

Dirac automaton: De Sitter covariance (non linear)

Covariance for Dirac QCA cannot leave m invariant

invariance of de Sitter norm:

$$\mathrm{Disp}(A): \quad \sin^2\omega - (1-m^2)|\mathbf{n}(\mathbf{k})|^2 - m^2 = 0$$

➡ $SO(1,4)$ invariance

$SO(1,4) \longrightarrow SO(1,3)$ for $m \to 0$ $\qquad \mathcal{O}(m^2)$

QUit quantum information theory group

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INFN Istituto Nazionale di Fisica Nucleare

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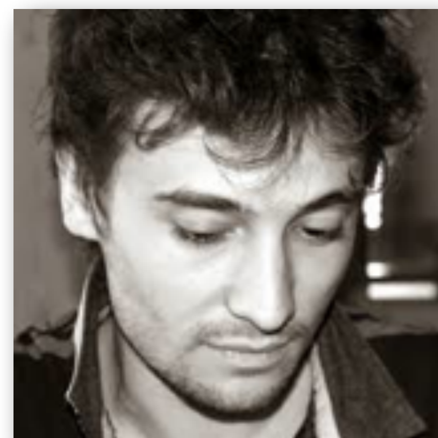

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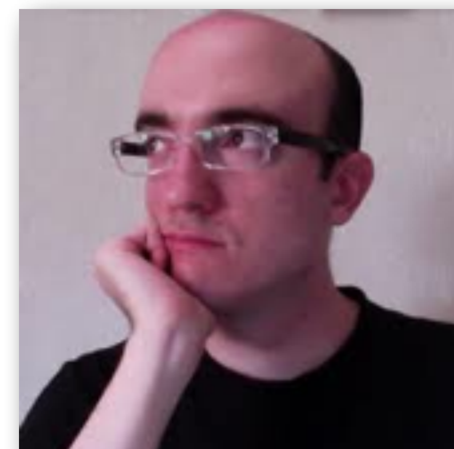

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