A COMPUTATIONAL GUT

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WHY QUANTUM?

QFT?

RELATIVITY?

Is there something more than Quantum?

Quantization rules, $\hbar$?

POSSIBLE ANSWER: THE UNIVERSE IS A HUGE QUANTUM COMPUTER

HOW RELATIVITY EMERGES FROM THE COMPUTATION?

Lorentz transformations from Galileo principle

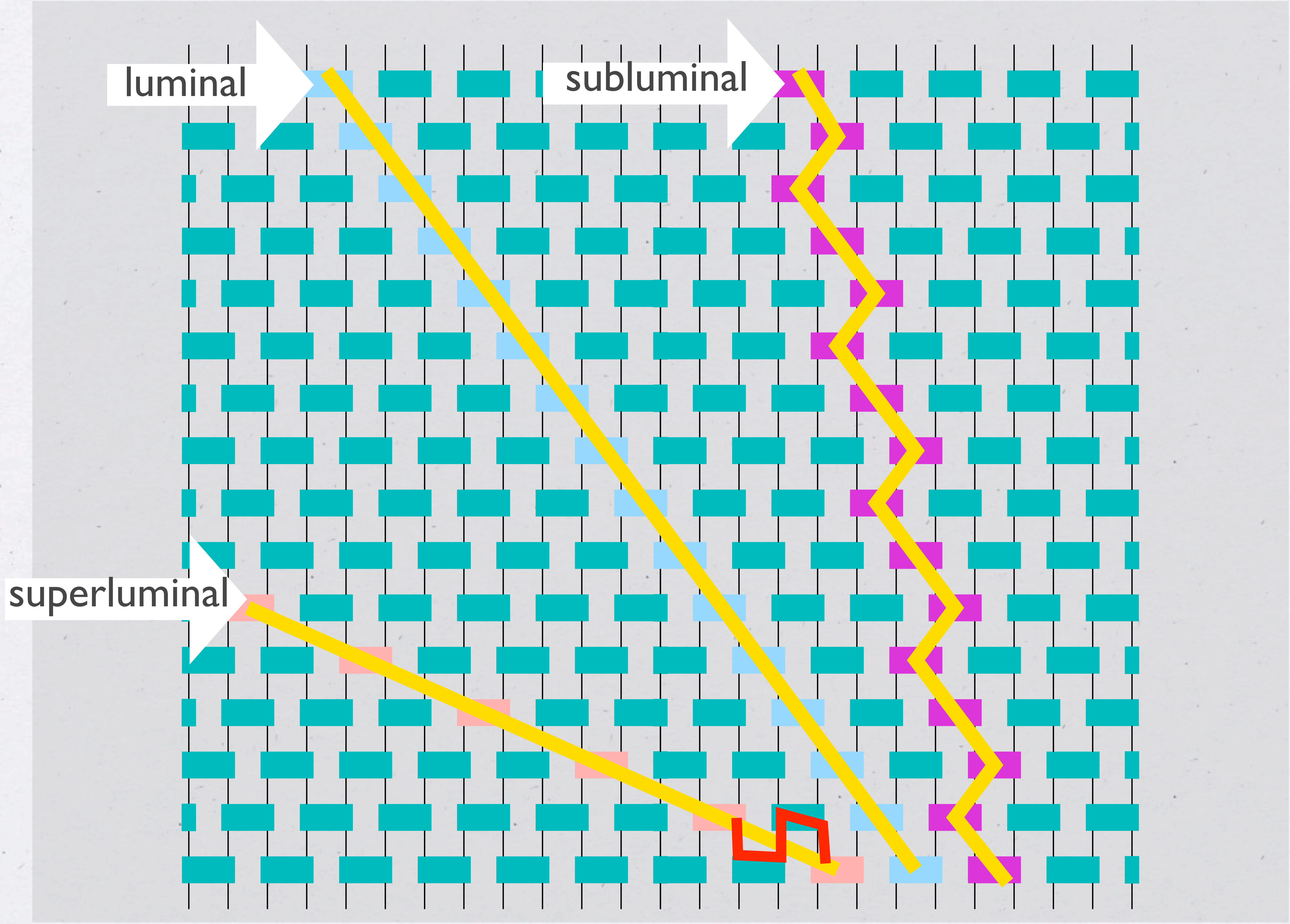
* Galileo principle includes homogeneity and isotropy of space and homogeneity of time.

* On the assumption of isotropy and homogeneity of space and homogeneity of time along with symmetry between the two references, the most general transformations of reference system are the Lorentz transformations with a parameter Ω with the dimensions of a velocity, which is independent on the relative velocity of frames.

* Empirically $\Omega=c$, which is an upper bound for velocities. ■

Special Relativity from computational network

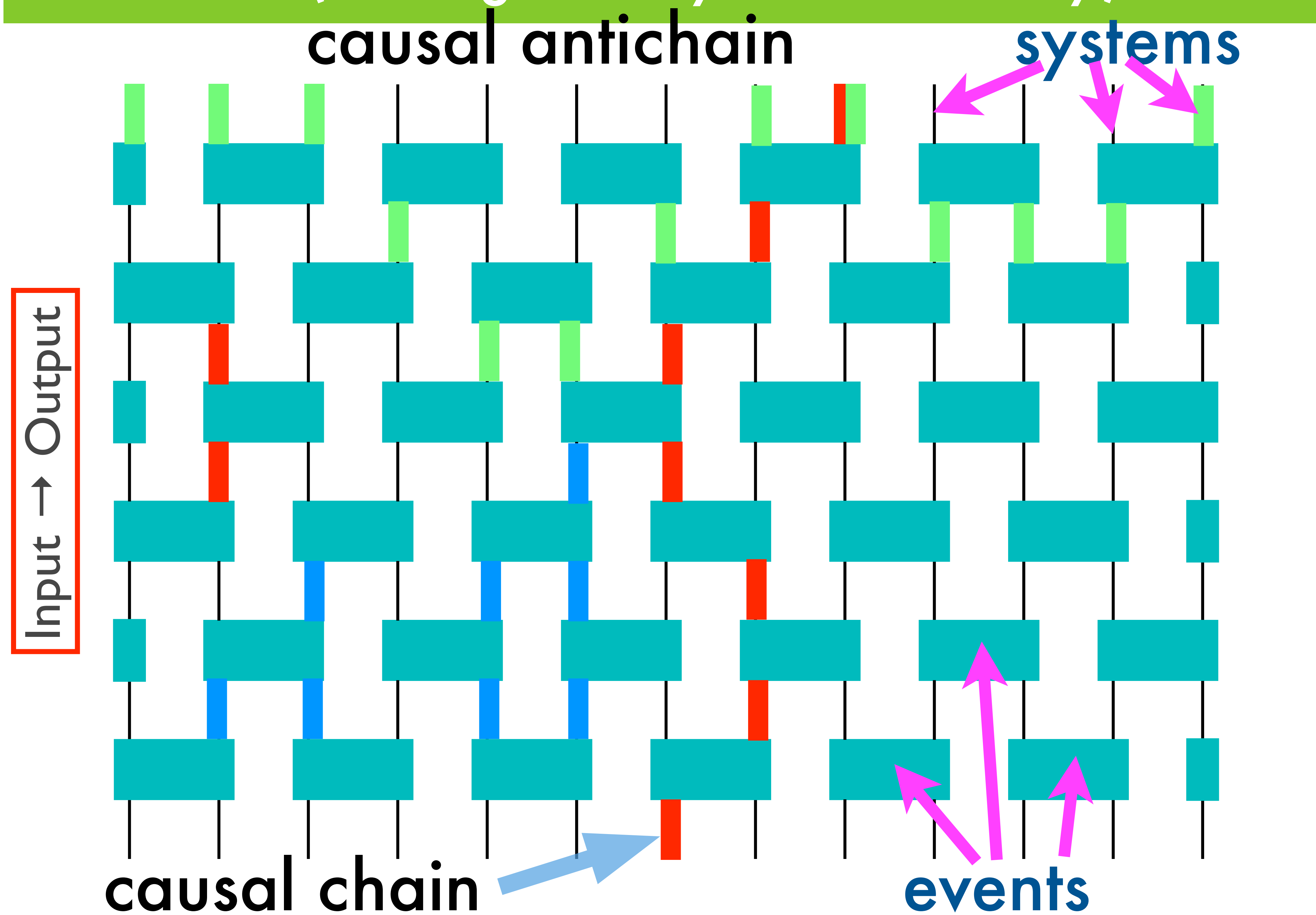
- Take a computational circuit which is uniform and isotropic.
- Take the "continuum limit" → space-time.
- Take only finite-system gates → bound on speeds ■.


luminal
subluminal
superluminal

Relativity from QT

(more generally from causality)

topology
no metric
only event-counting



causal immediateness

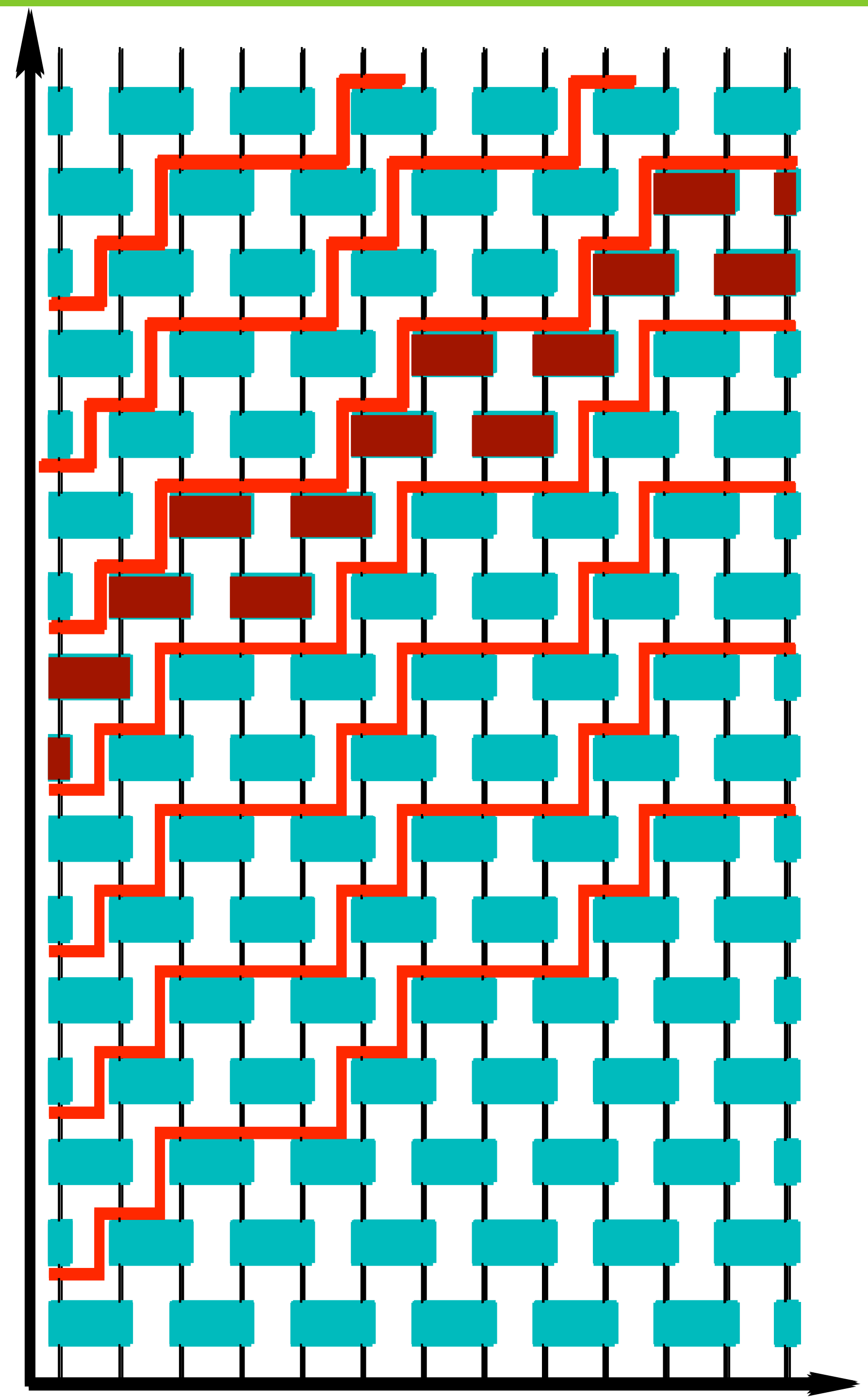
causal propinquity

slice

chain (time)

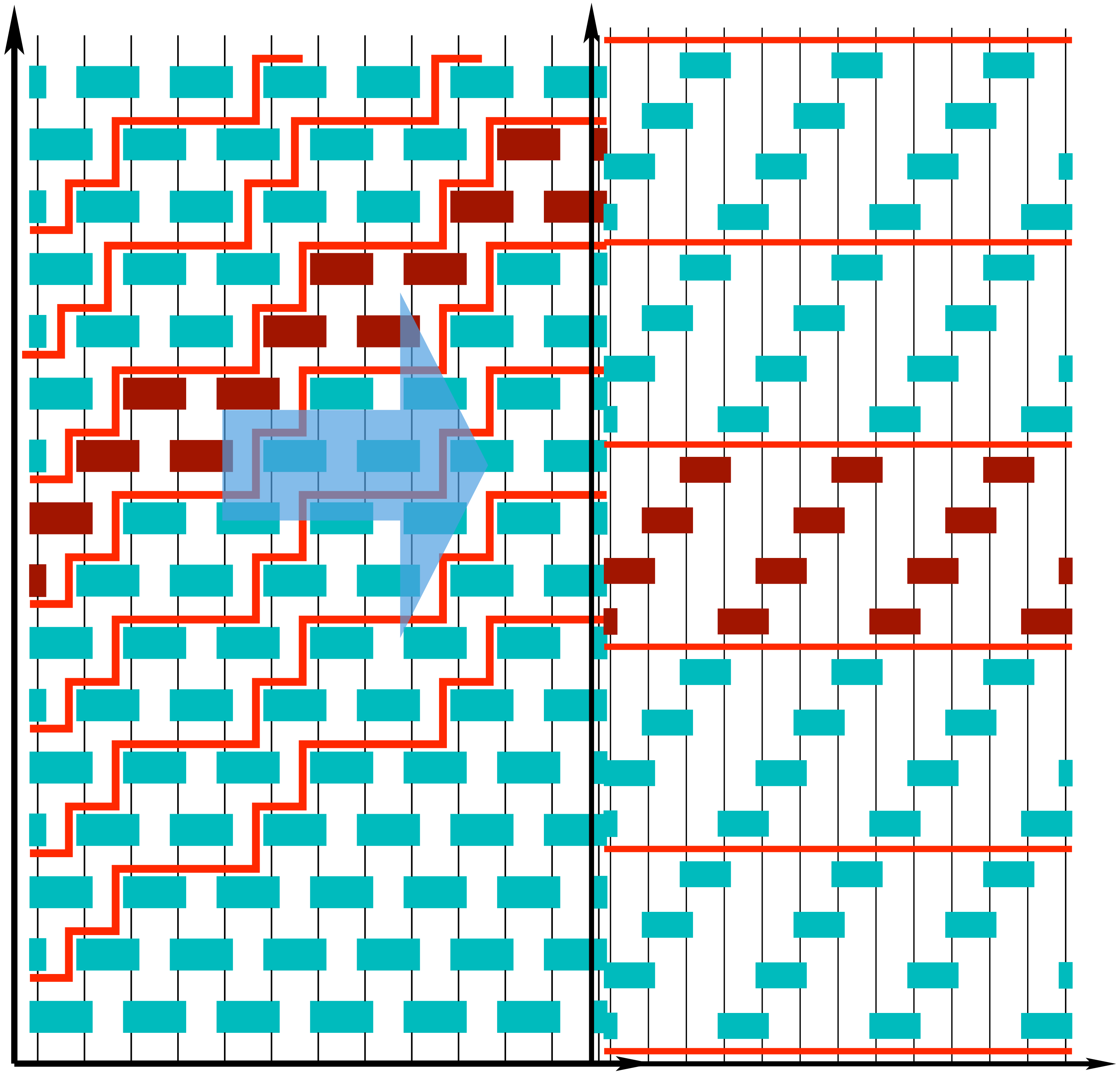
antichain (space)

Relativity from QT



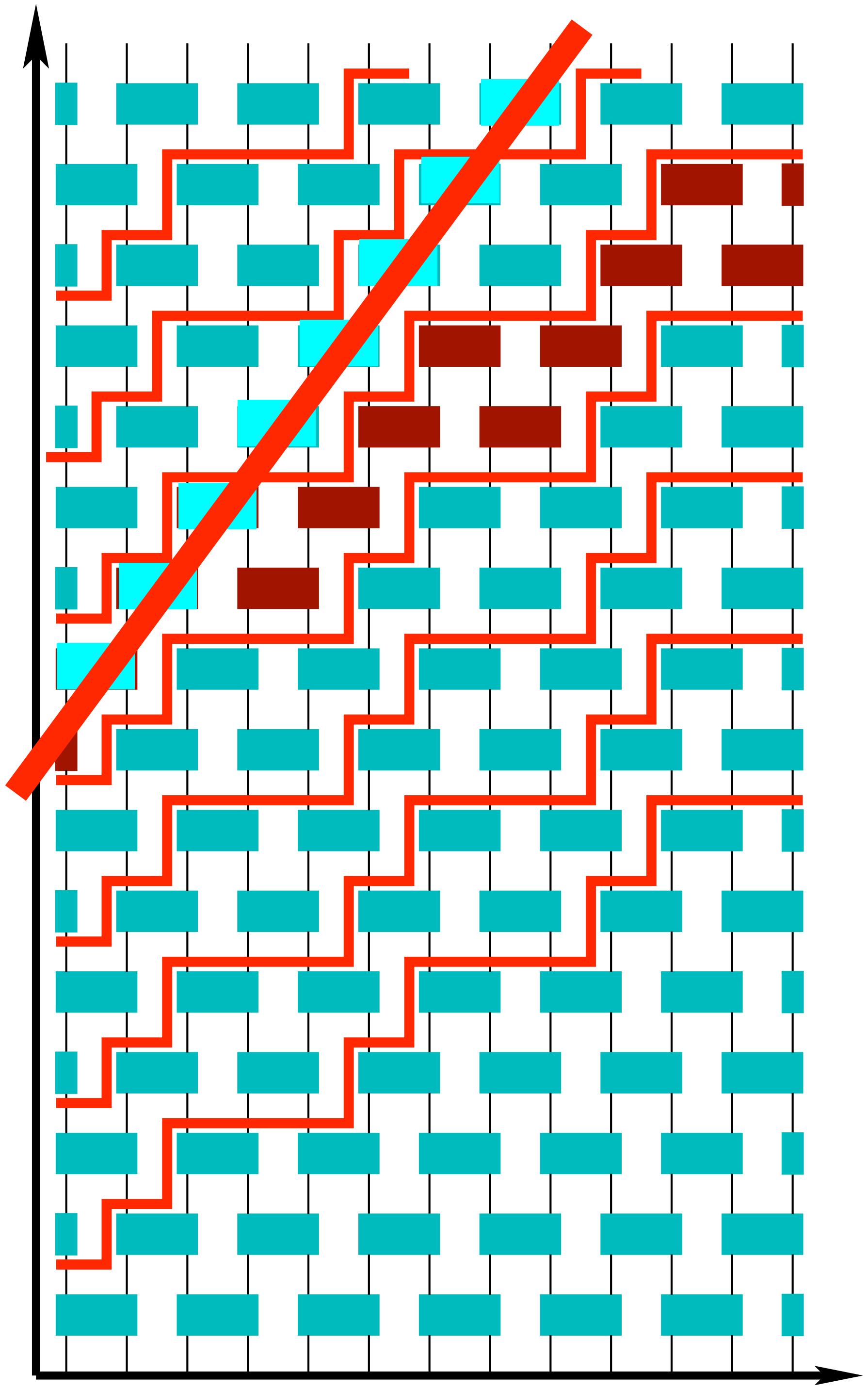
build a uniform foliation

Relativity from QT



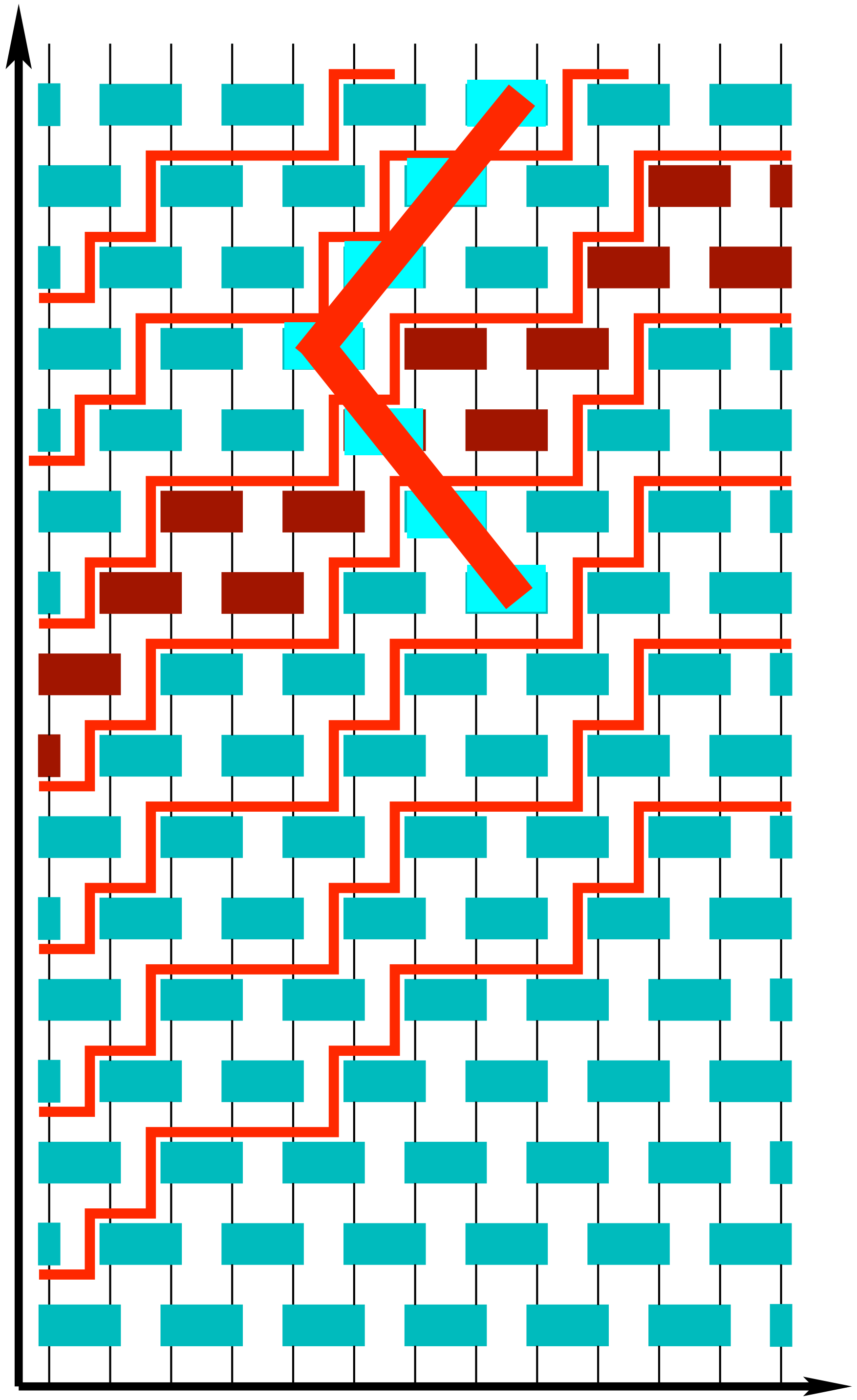
change reference

Relativity from QT



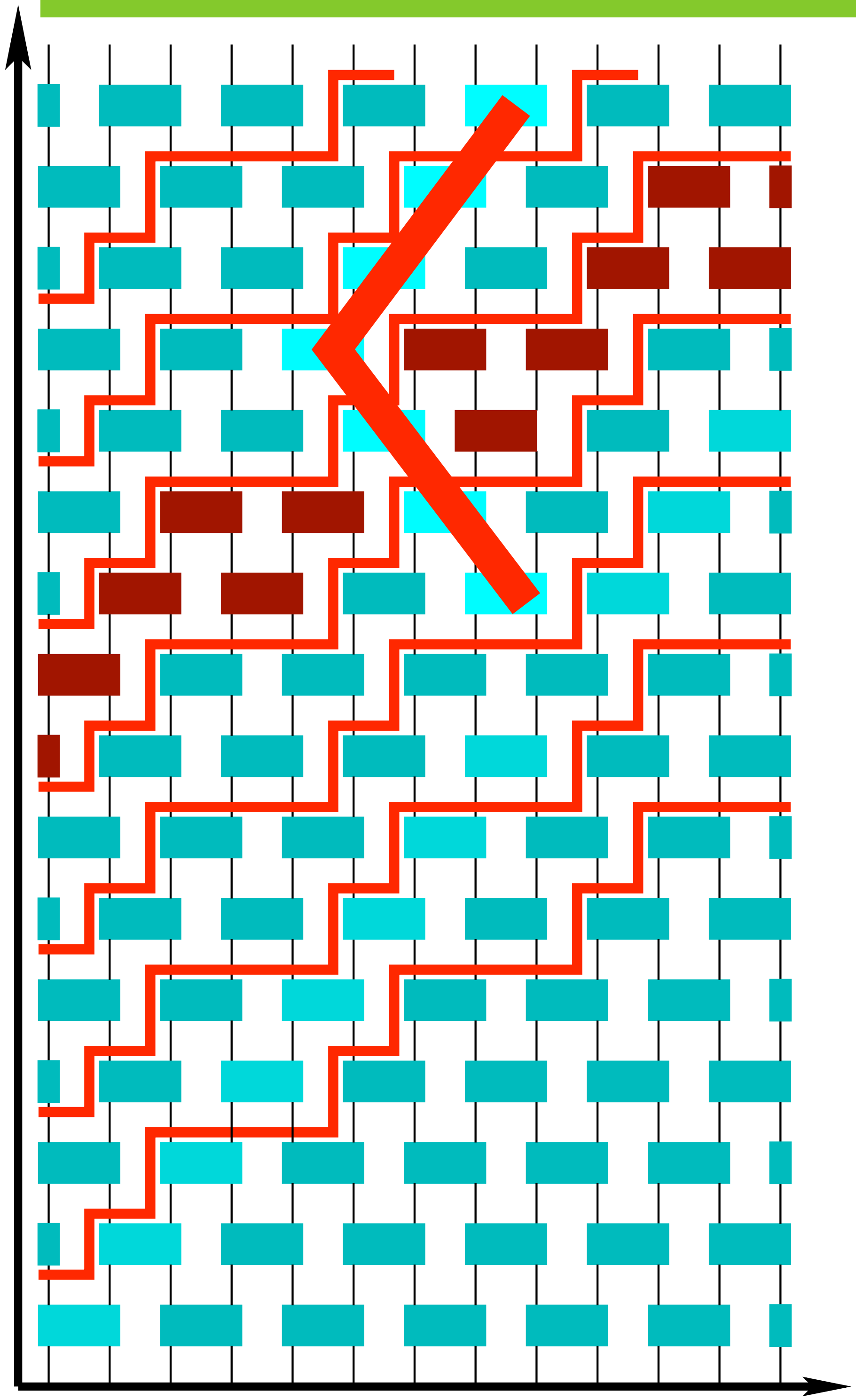
speed of light

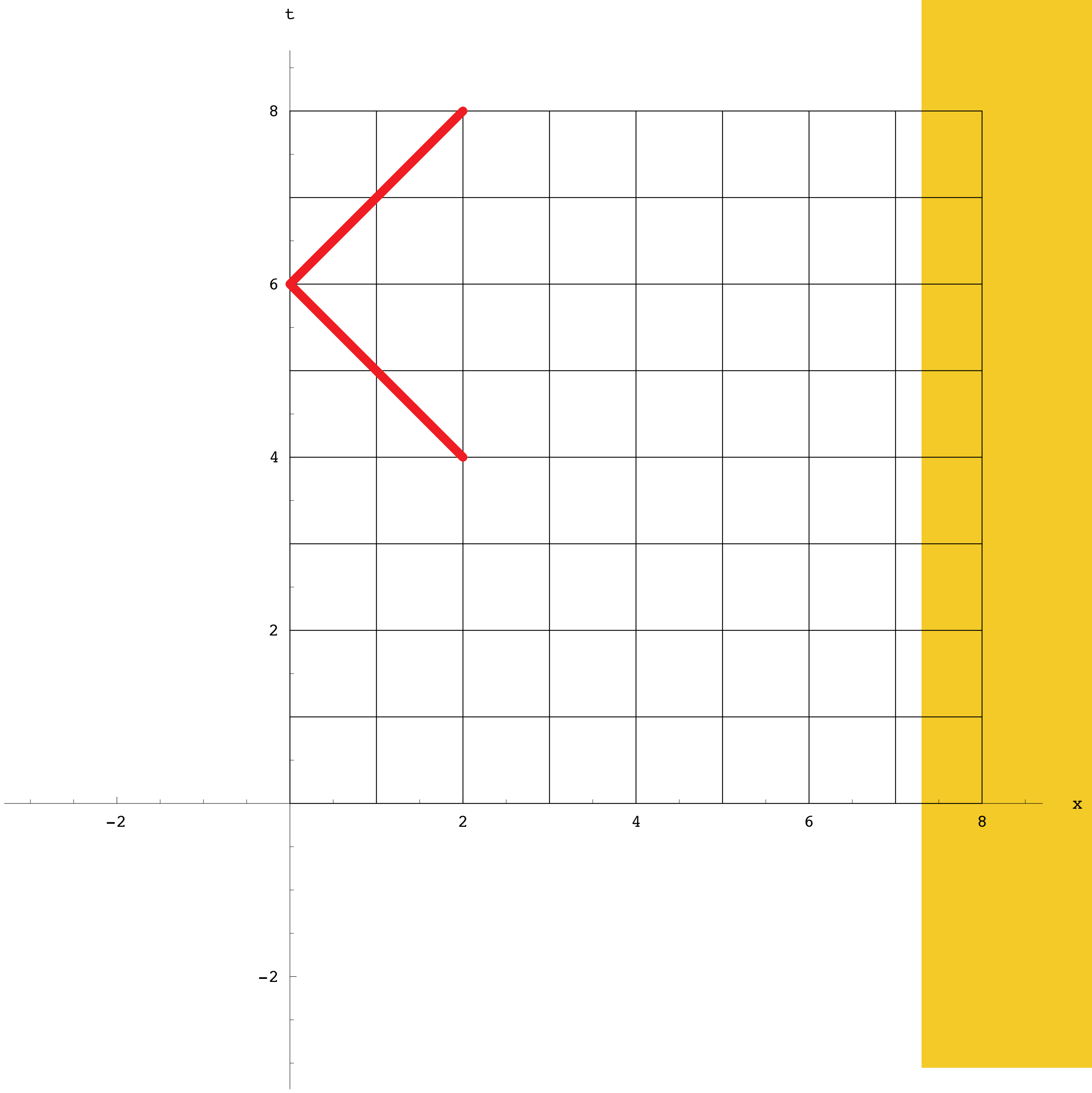
Relativity from QT



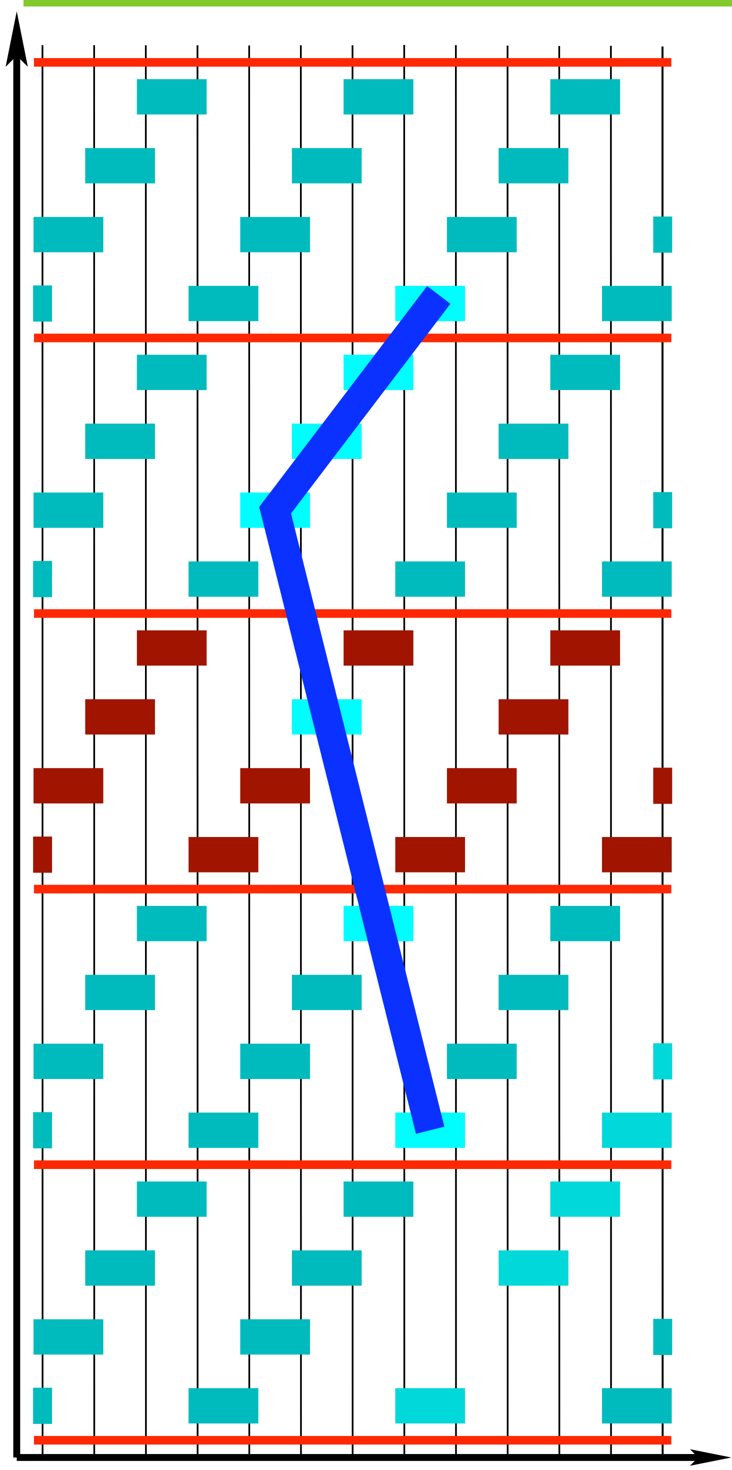
clock tic-tac

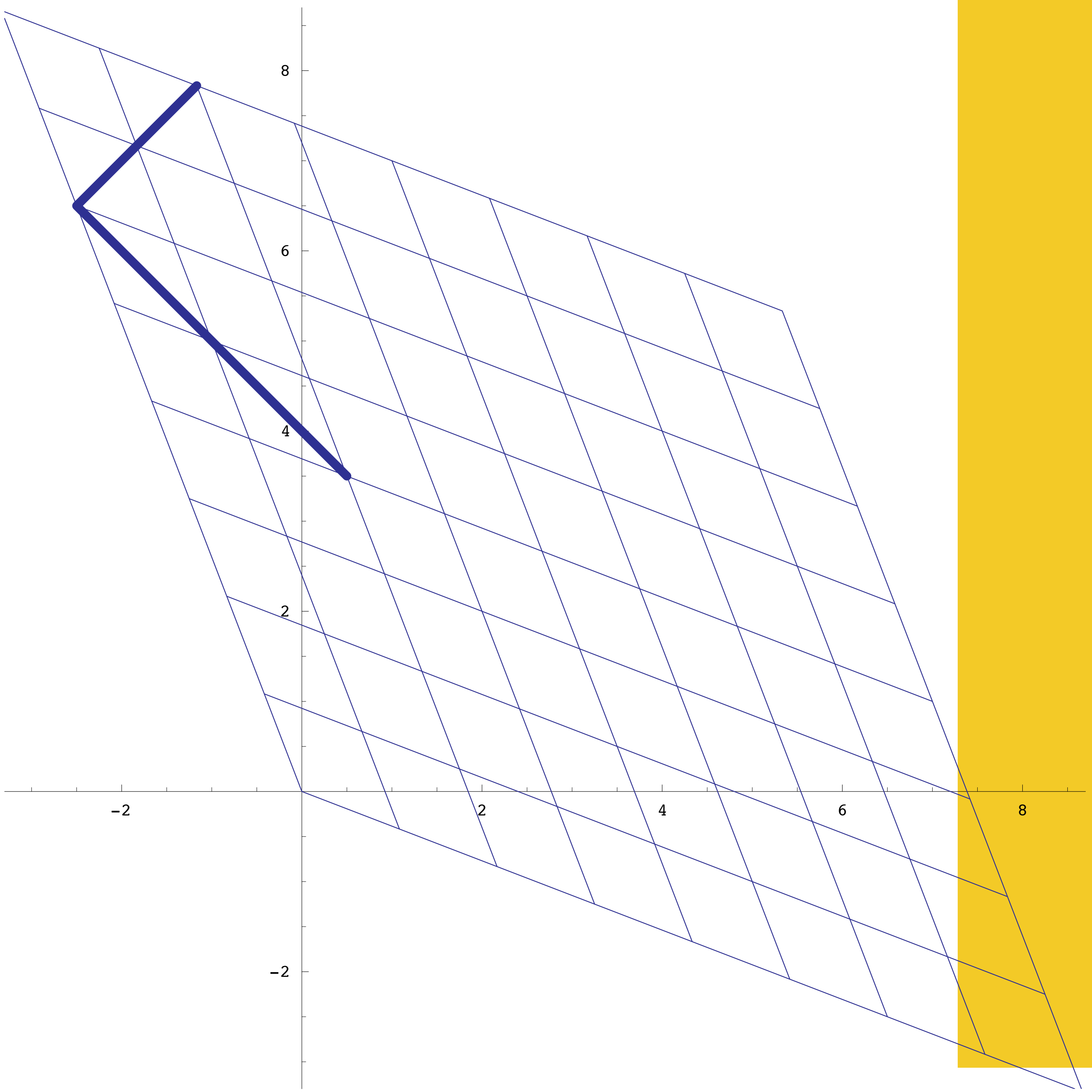
Relativity from QT

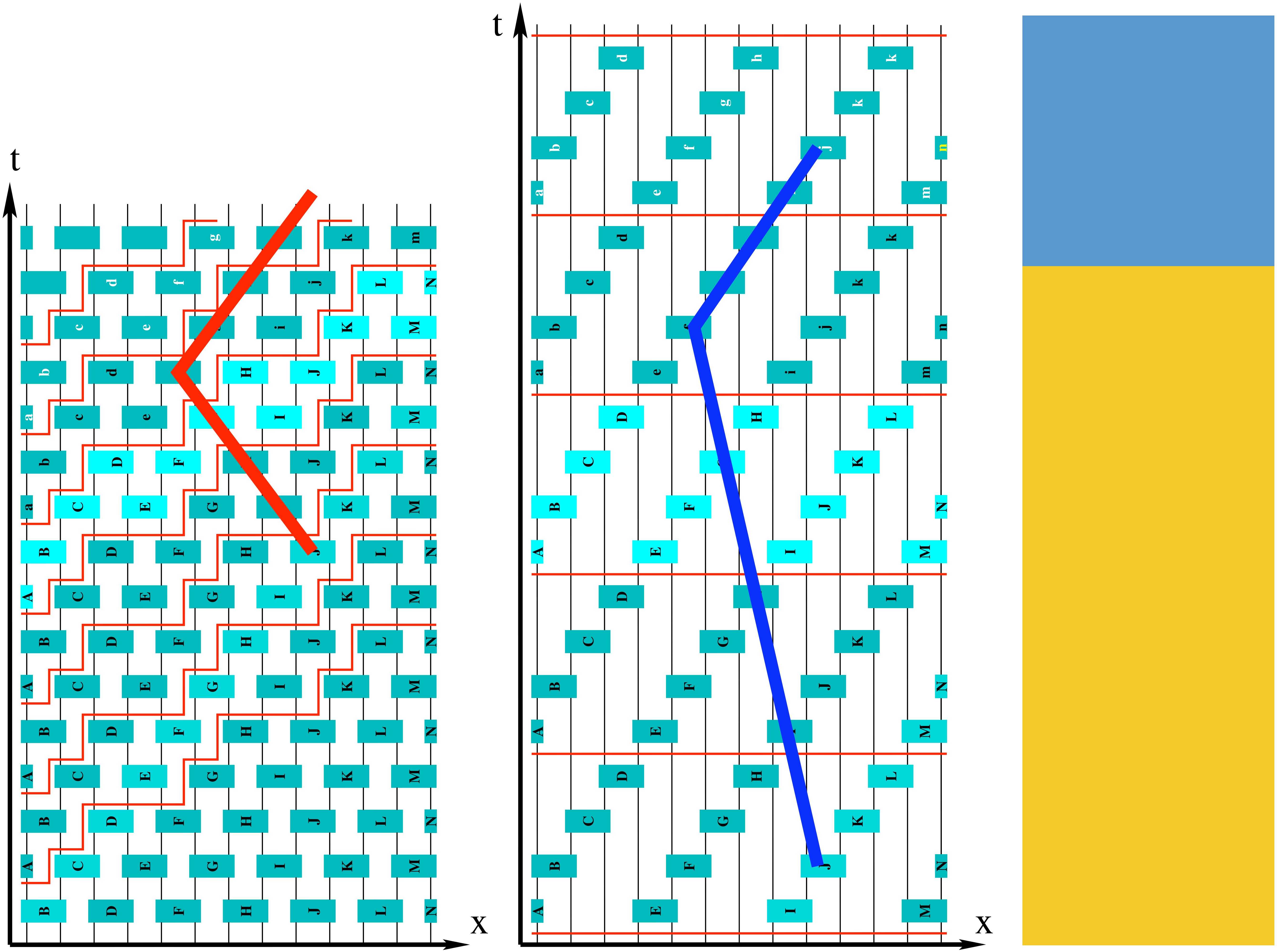




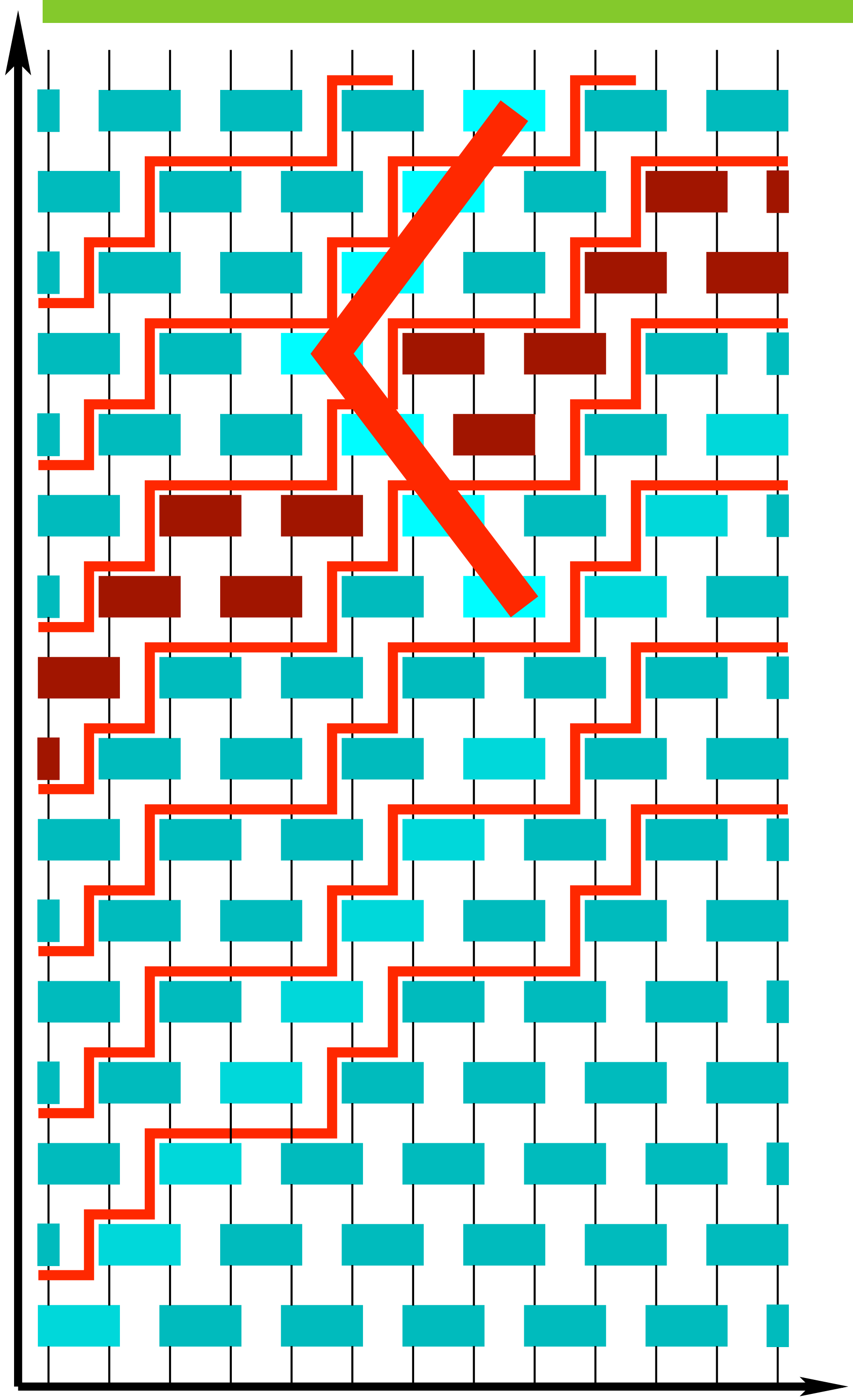
Relativity from QT

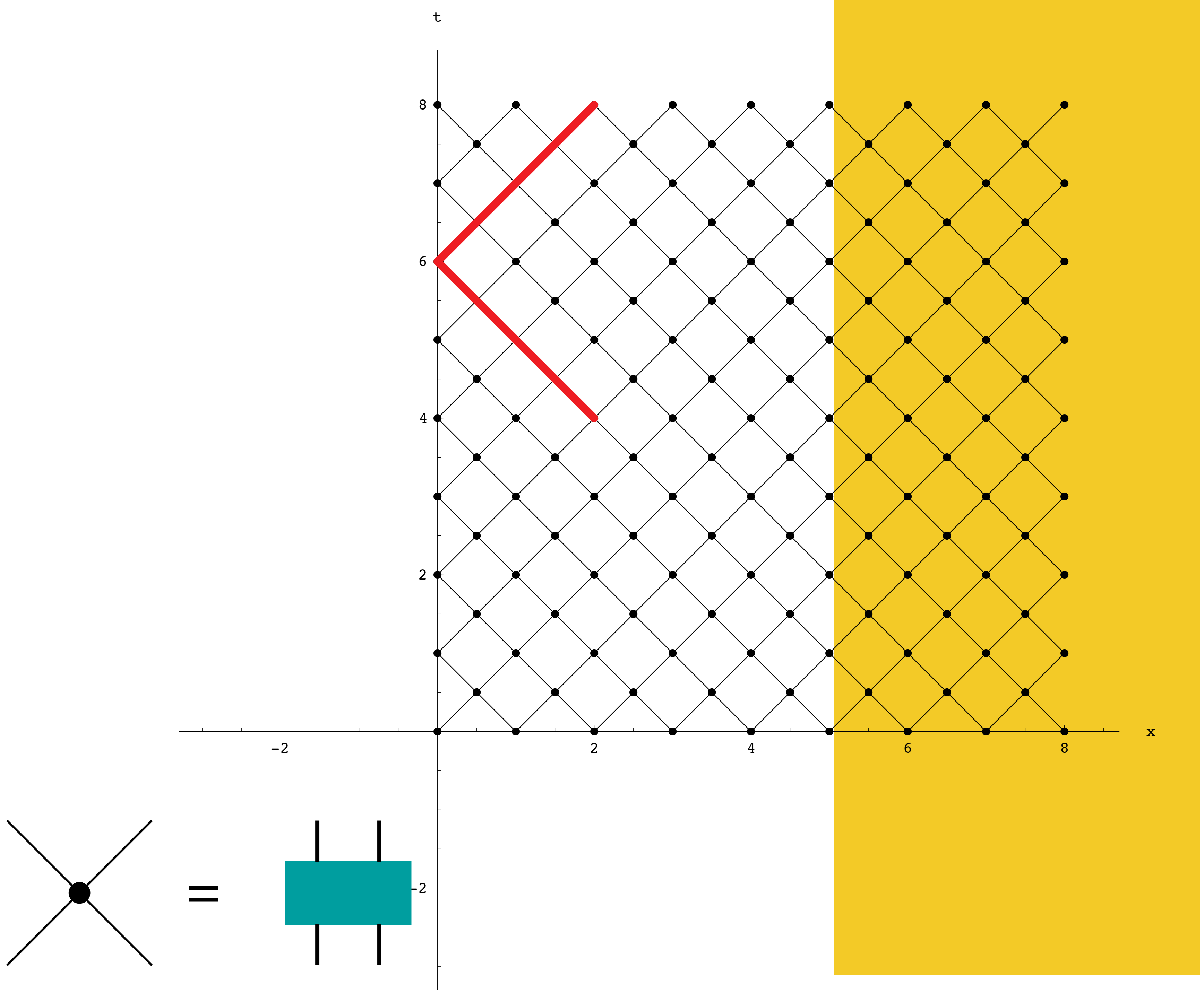




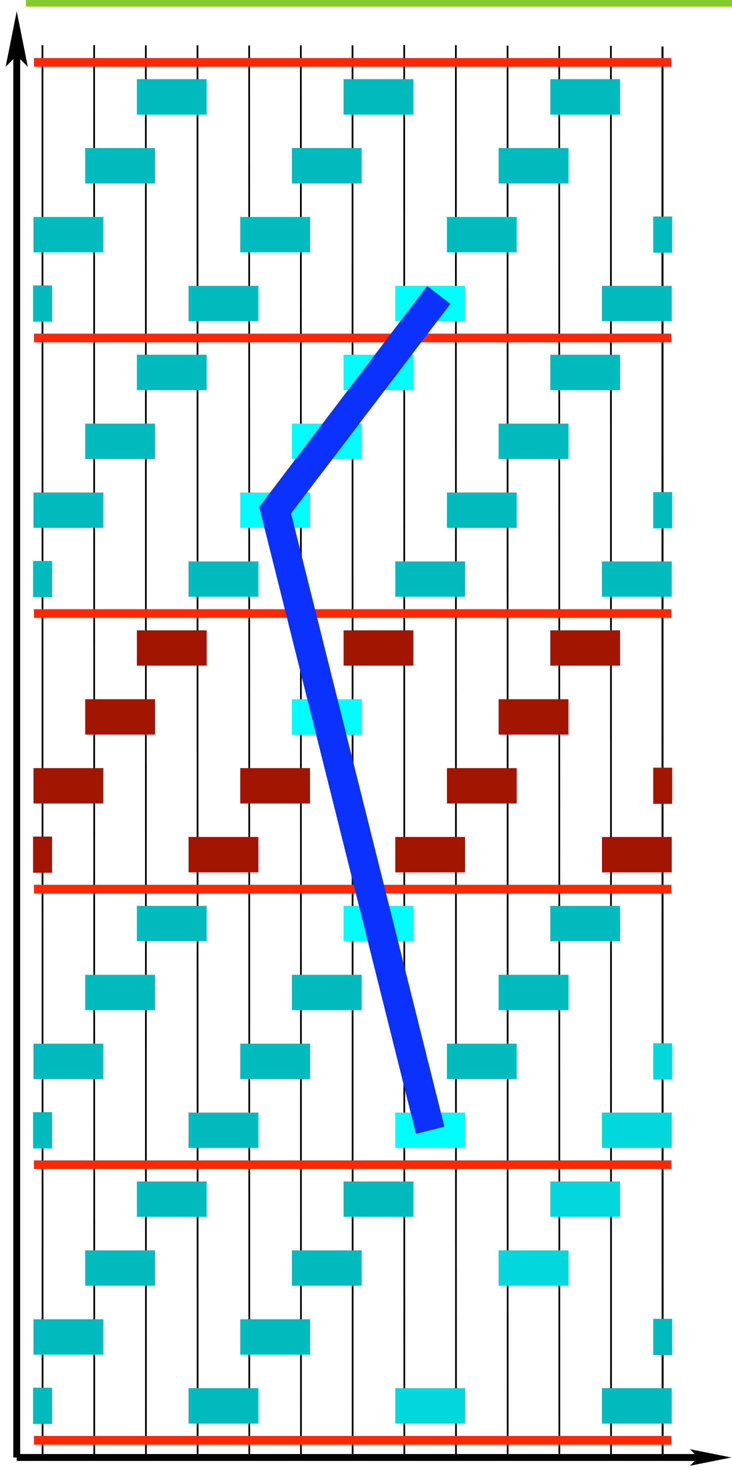

t
x
t
x

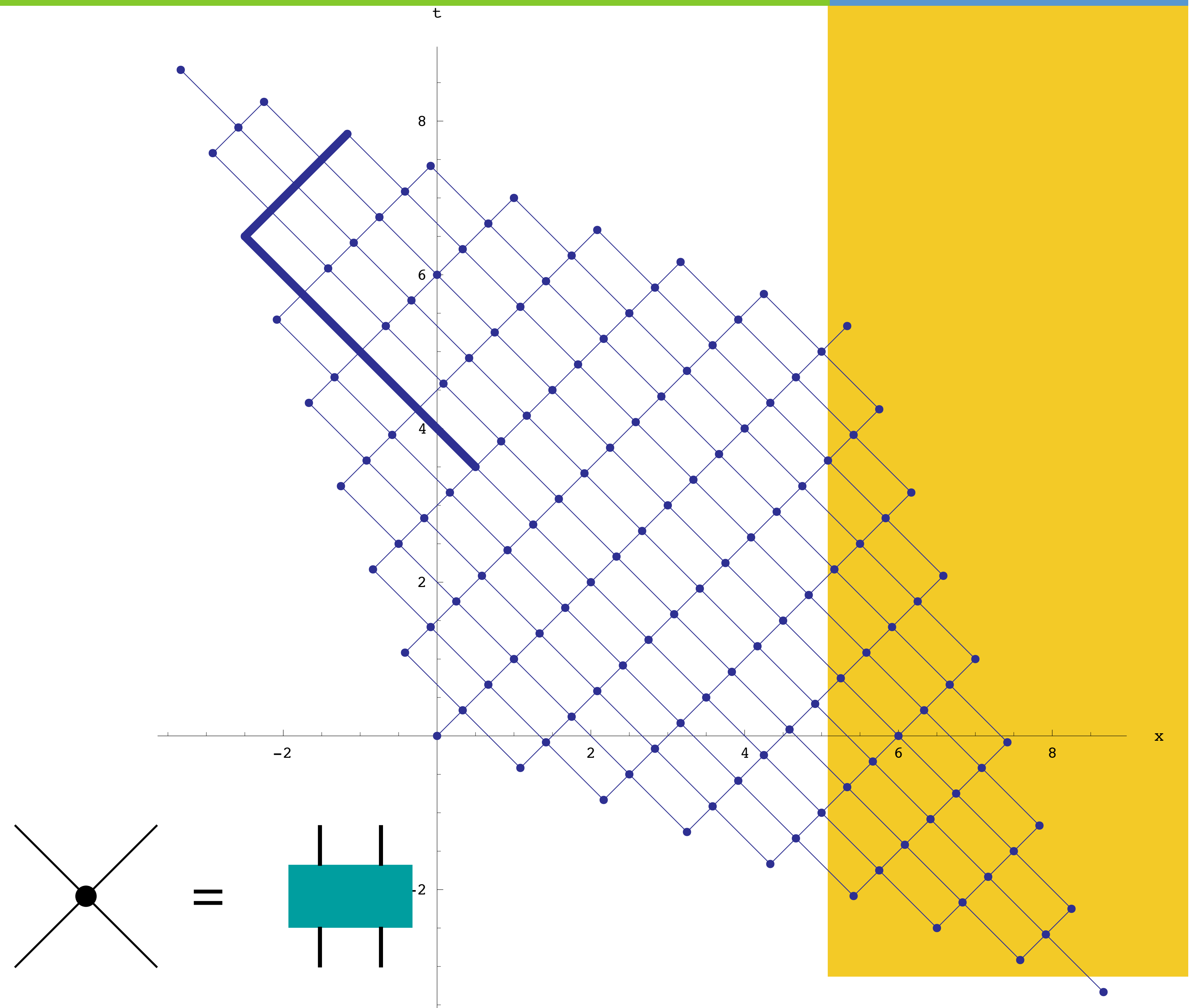
Relativity from QT



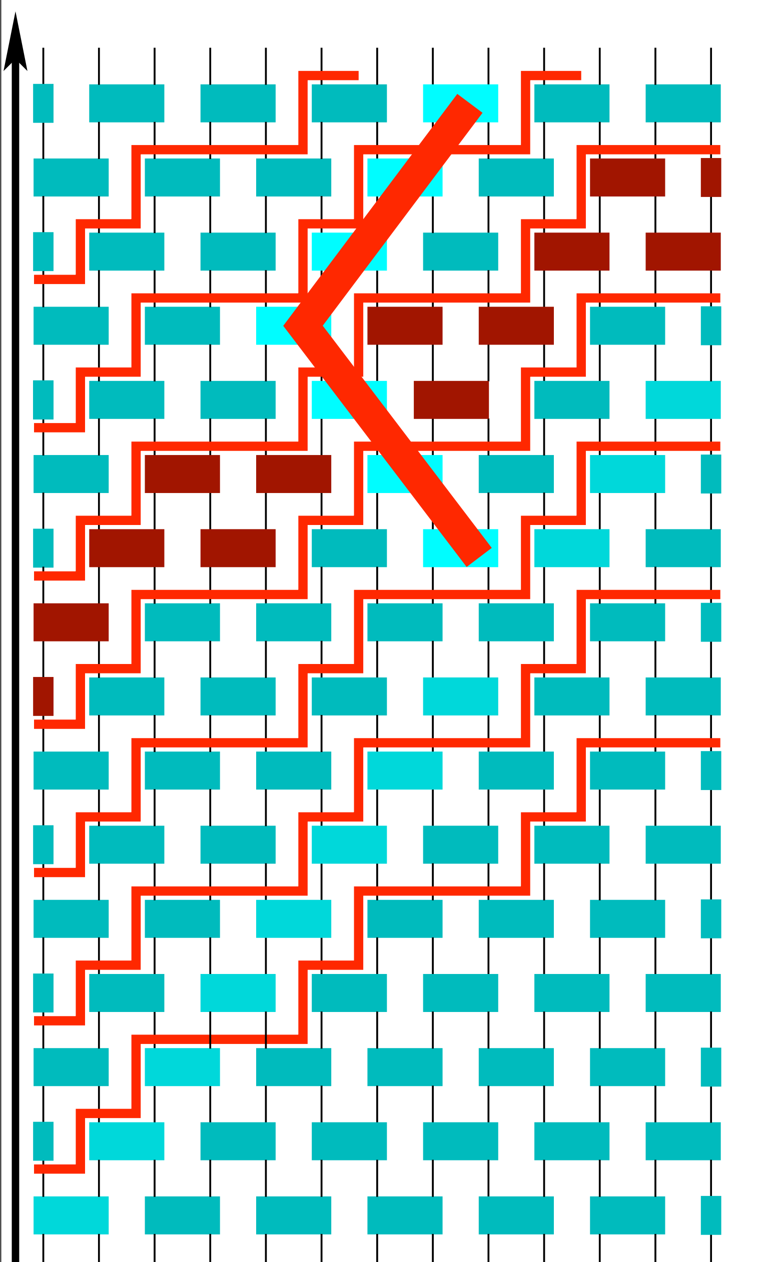


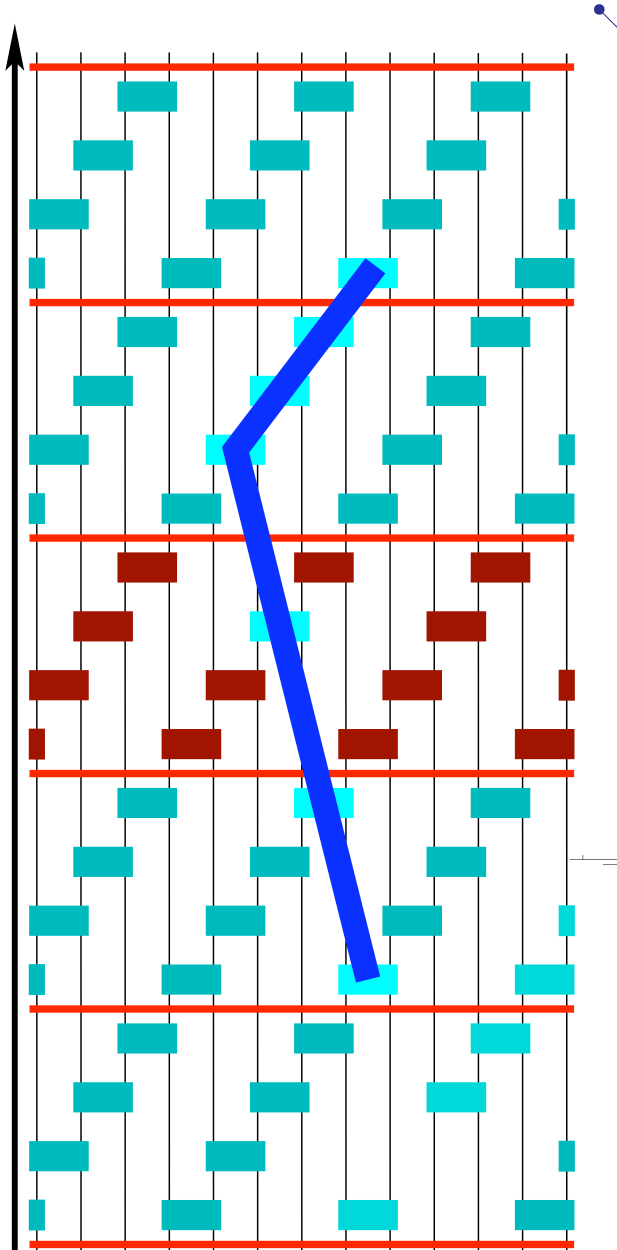
Relativity from QT

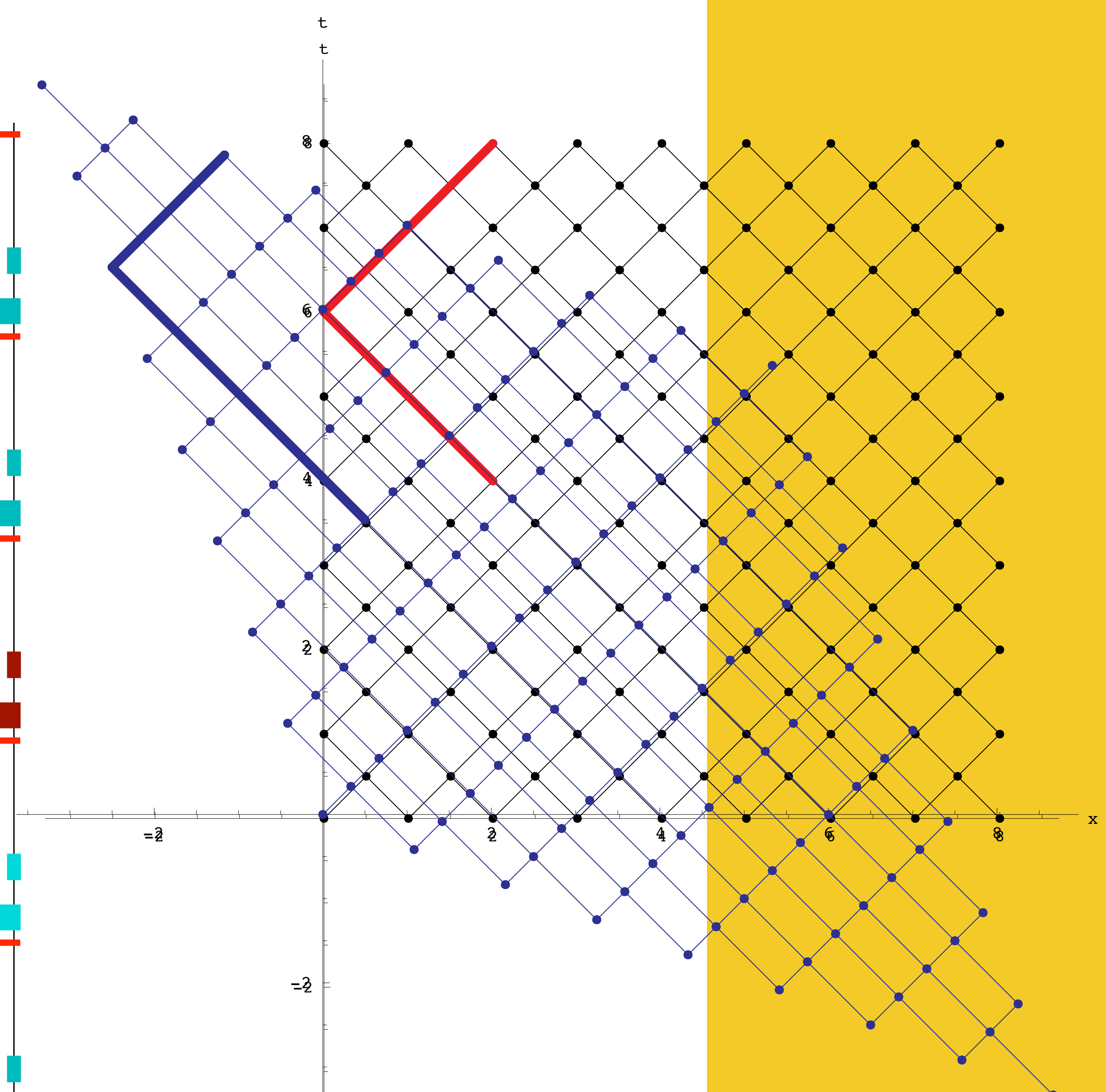




Relativity from QT





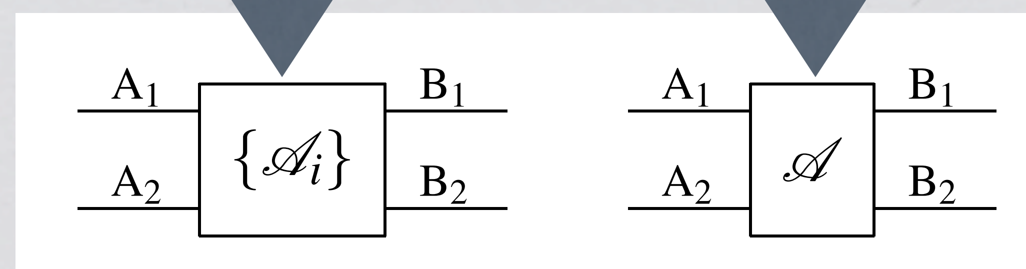


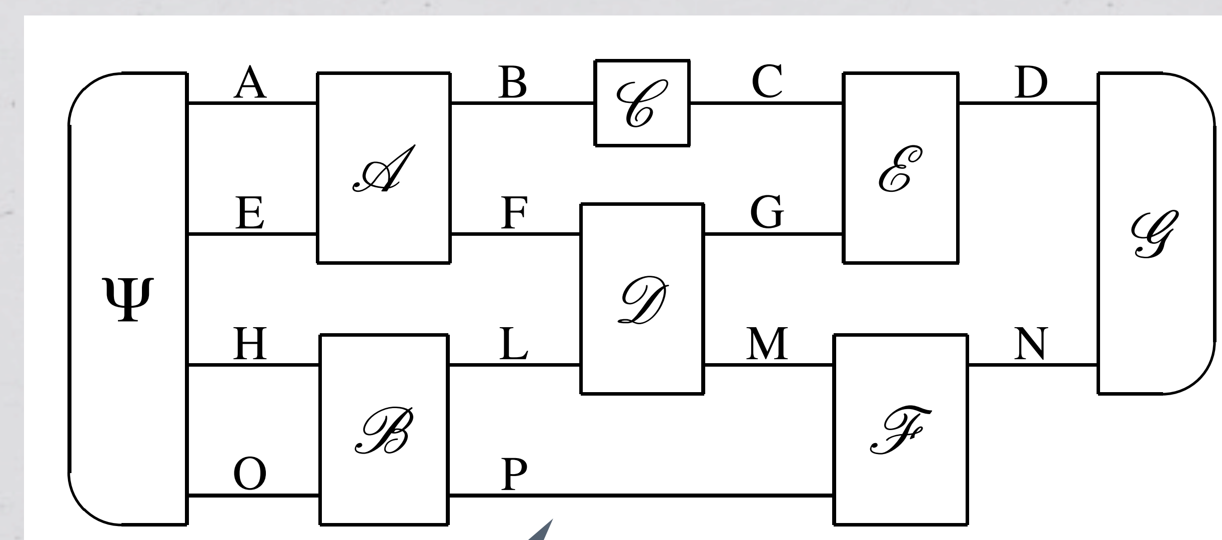
WE GOT SR FROM PURE CAUSALITY!

The Operational Framework

test

event





no loops

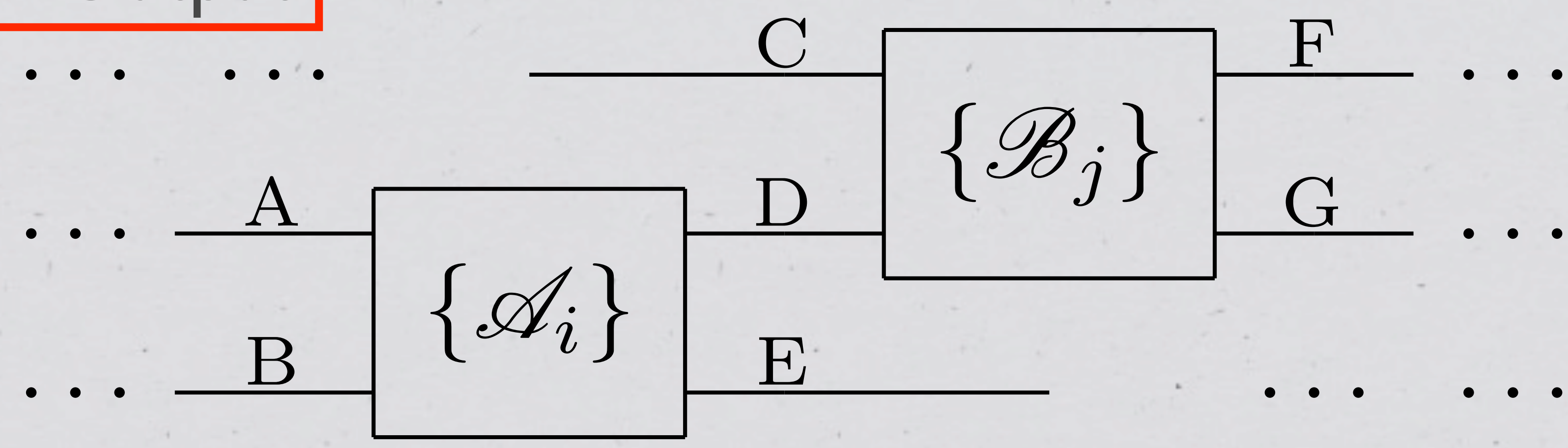
DAG (directed acyclic graph)

- **Probabilistic operational theory:** every test from the trivial system to the trivial system is associated to a probability distribution of outcomes.

Causal probabilistic theories

Input → Output

DAG



A theory is *causal*, if for any two tests that are connected the marginal probability of the input event is independent on the choice of the output test, whereas, viceversa the marginal probability of the output event generally depends on the choice of the input test.

Wittgenstein-ism

1 The world is all that is the case.

1.1 The world is the totality of facts, not of things.

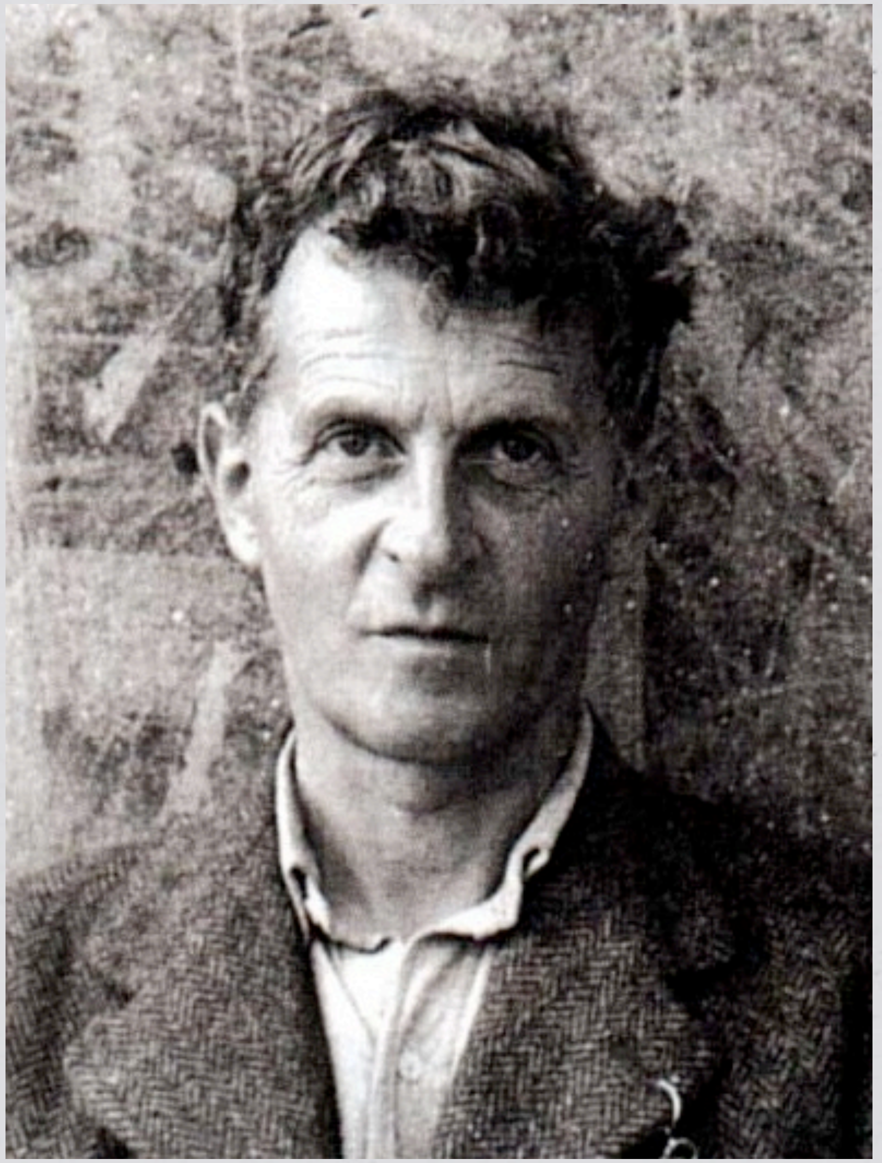
1.11 The world is determined by the facts, and by their being all the facts.

1.12 For the totality of facts determines what is the case, and also whatever is not the case.

1.13 The facts in logical space are the world.

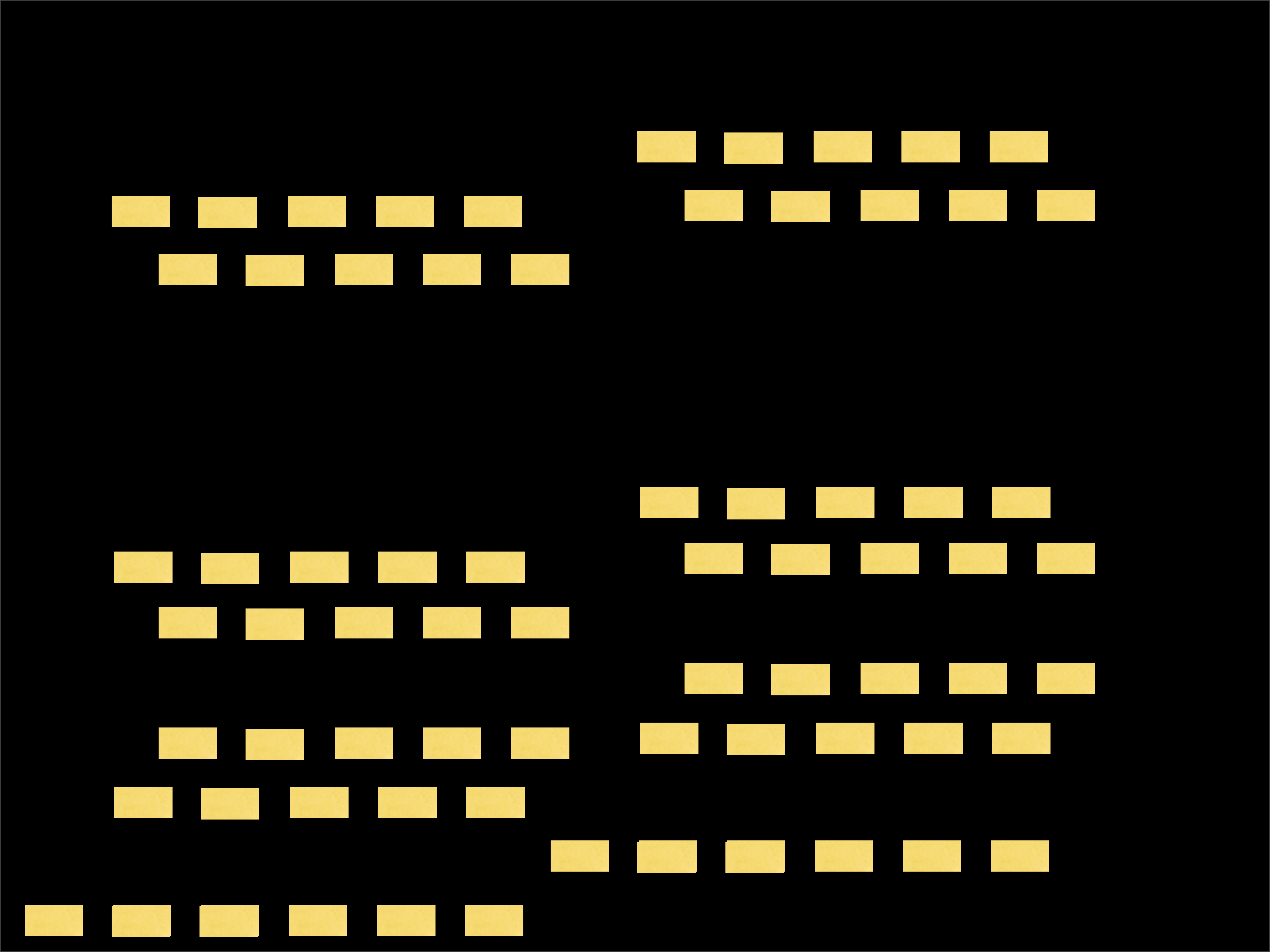
1.2 The world divides into facts.

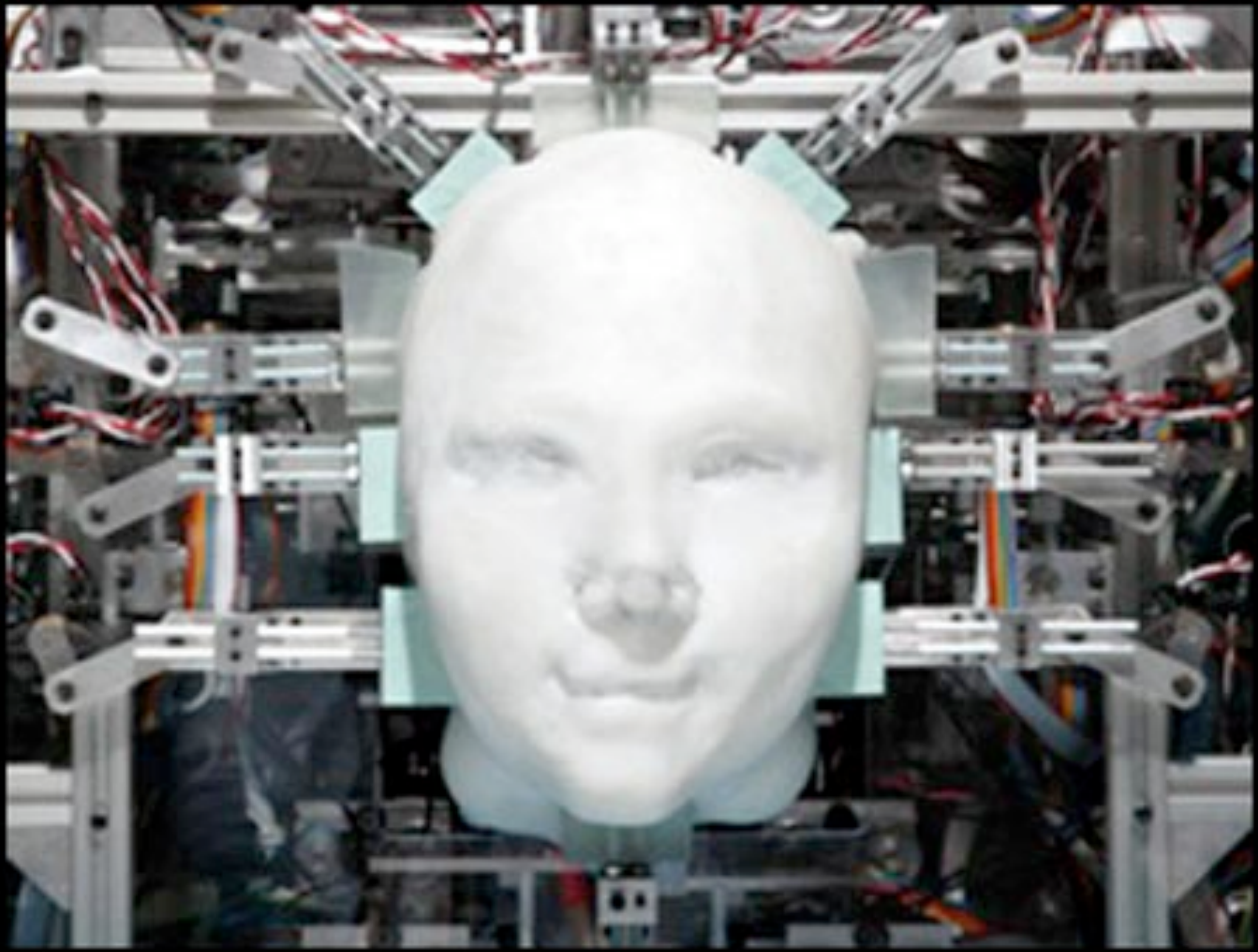
1.21 Each item can be the case or not the case while everything else remains the same.

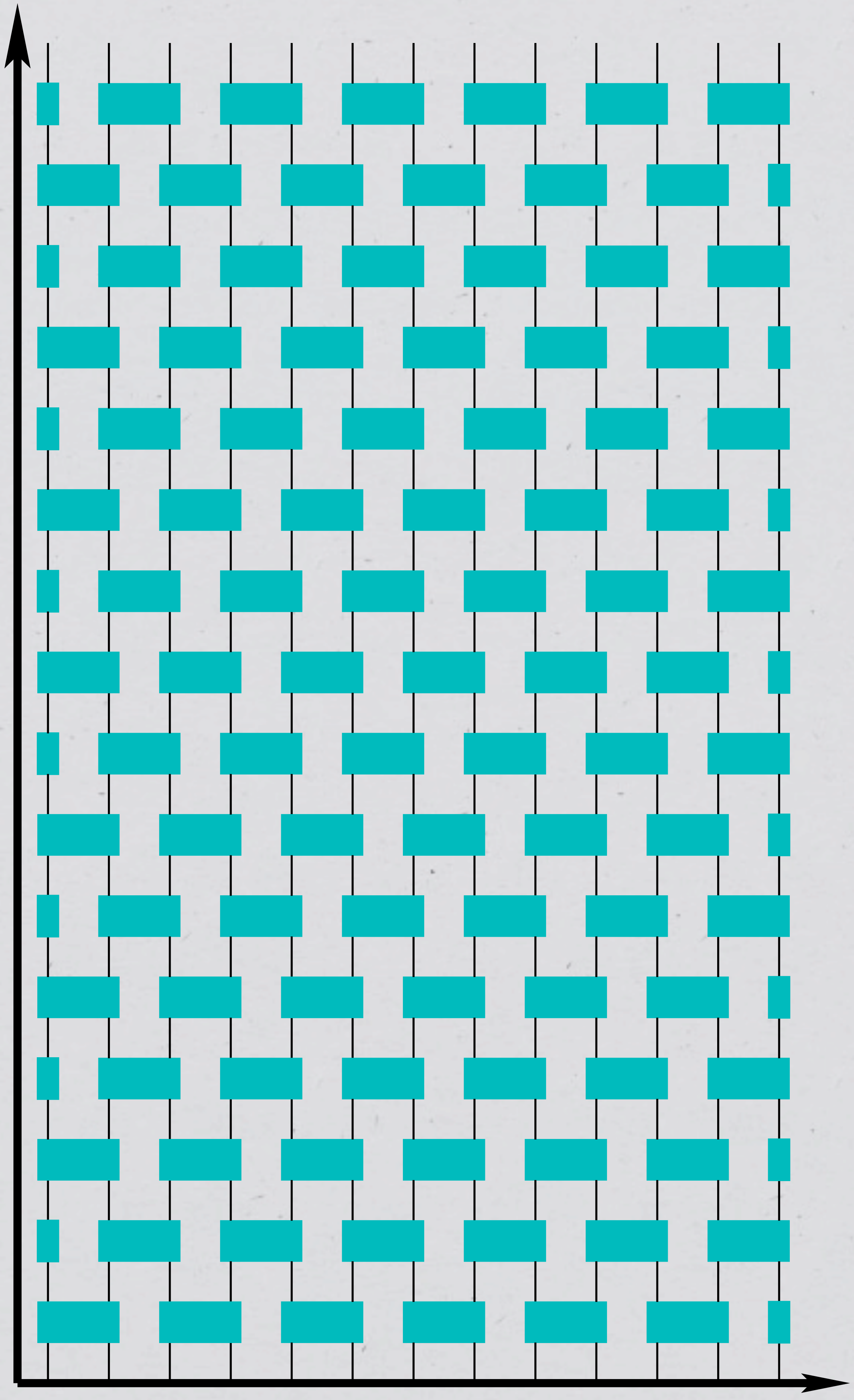


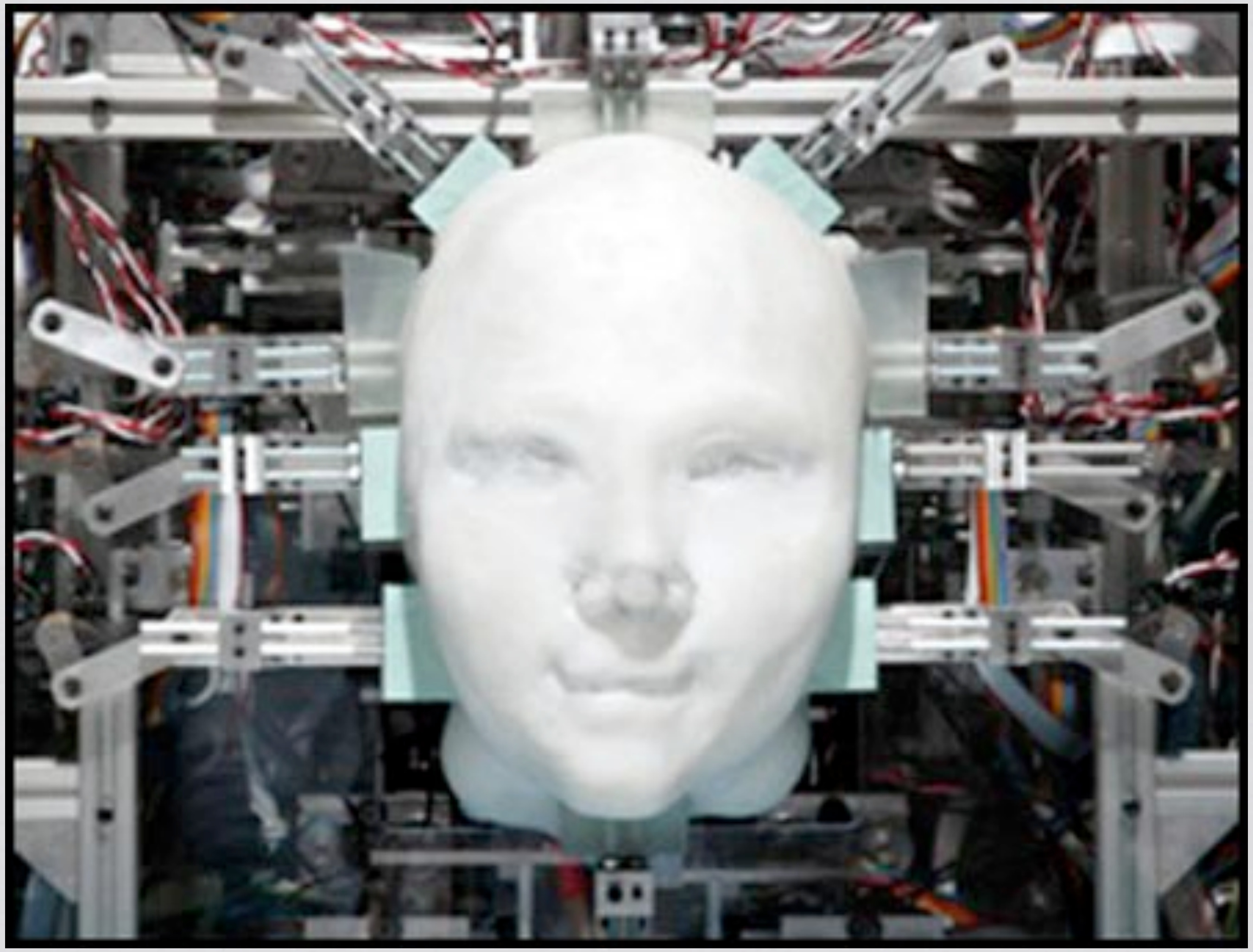
My Brief History of Space-Time

- At the beginning there were only events ...
  - Then the Man devised causal connections between them
    - He modeled the causal connections in a unified framework which is space-time





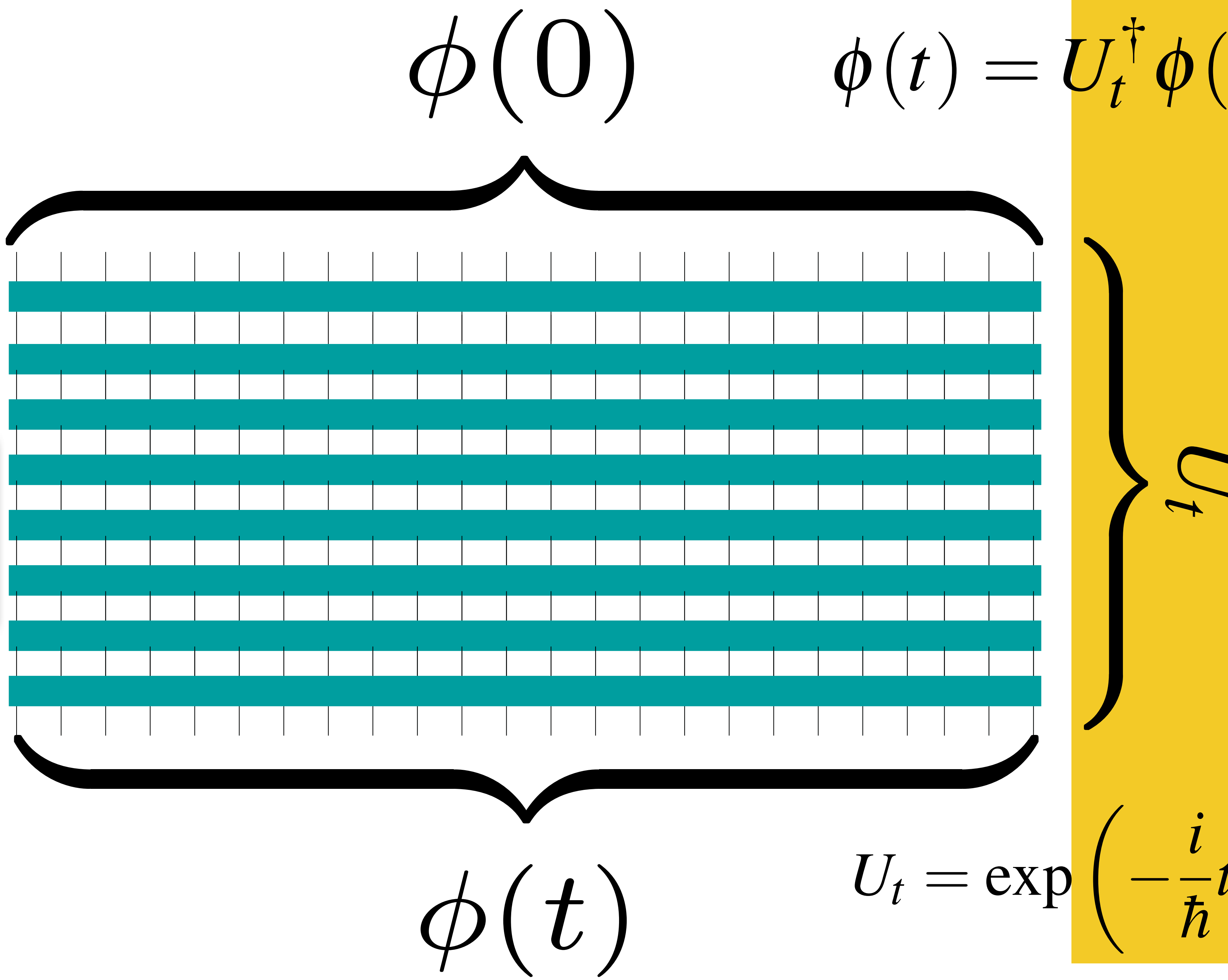




Quantum Computational Field Theory (QCFT)

$$\phi(t) = U_t^\dagger \phi(0) U_t$$

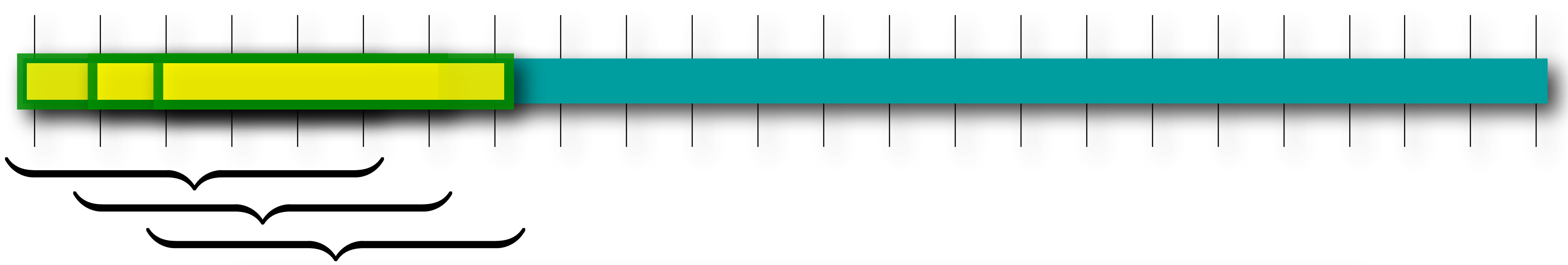
$$H = \sum_{<i,j>} H_{i,j}$$



$$U_t = \exp\left(-\frac{i}{\hbar} t\hbar\omega H\right)$$

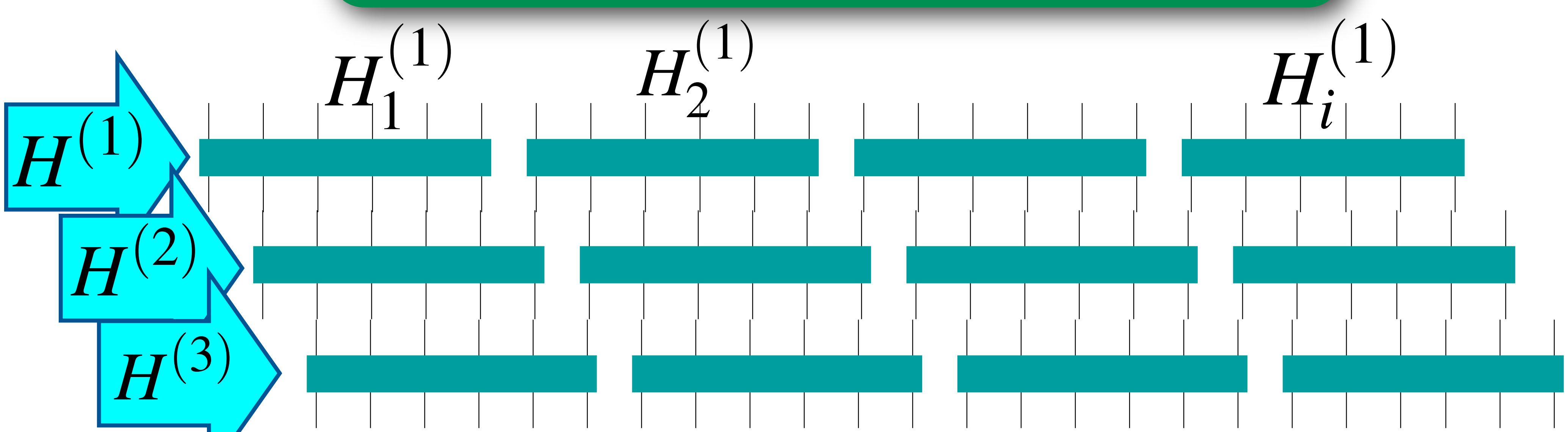
QCFT

p nn translational-invariant "Hamiltonian"



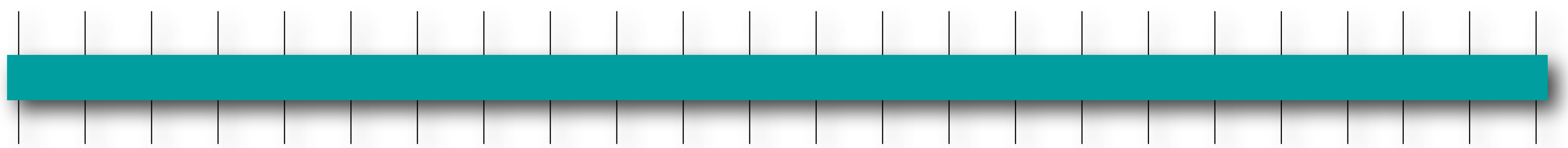
$$H = \sum_{k=0}^{p-1} H^{(k)}, \quad H^{(k)} = \sum_{i=-N_x}^{N_x} H_i^{(k)}$$

$$[H_i^{(k)}, H_j^{(k)}] = 0, \quad [H^{(l)}, H^{(k)}] \neq 0$$

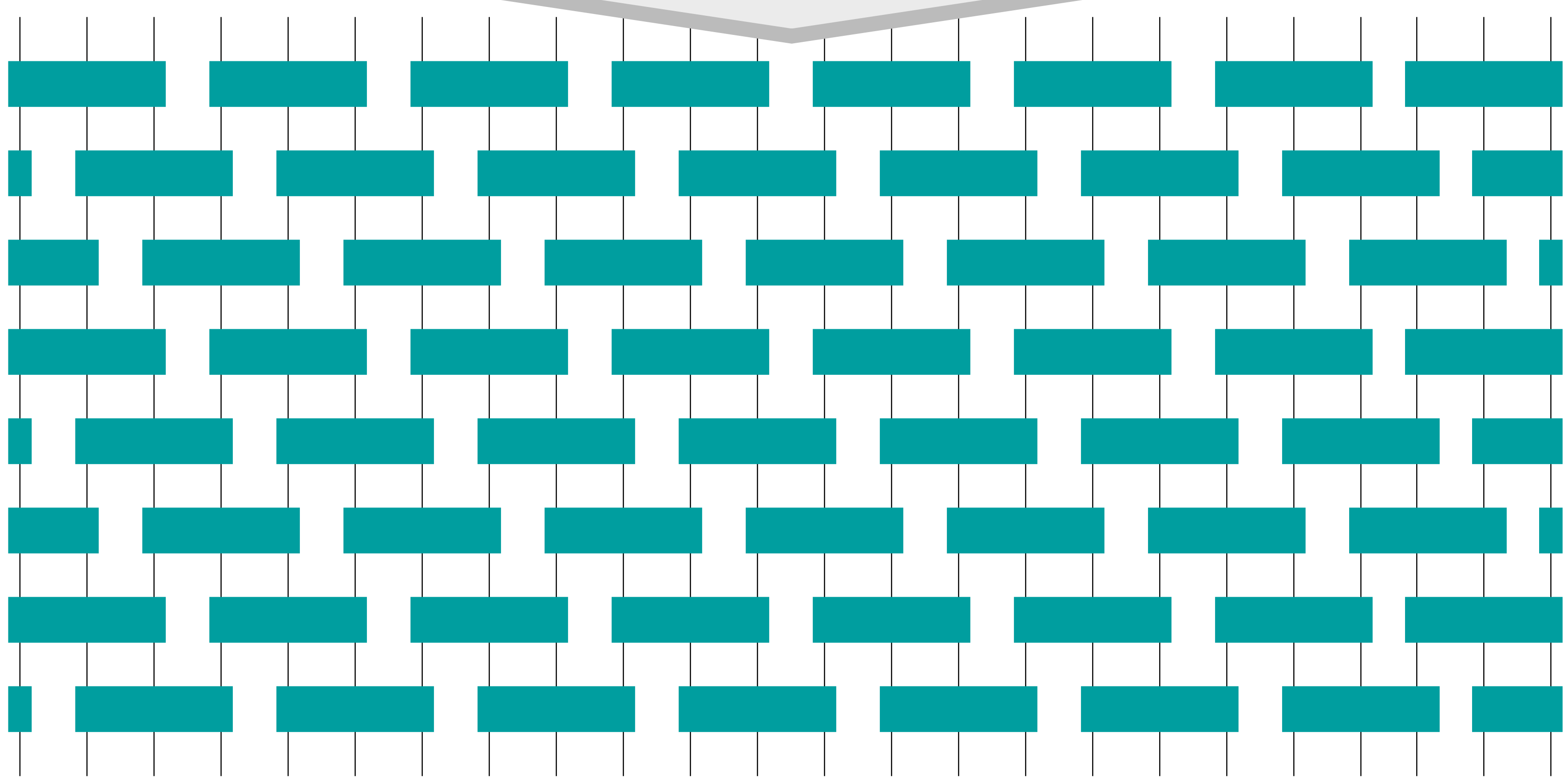


Trotter-ization of H

QCFT



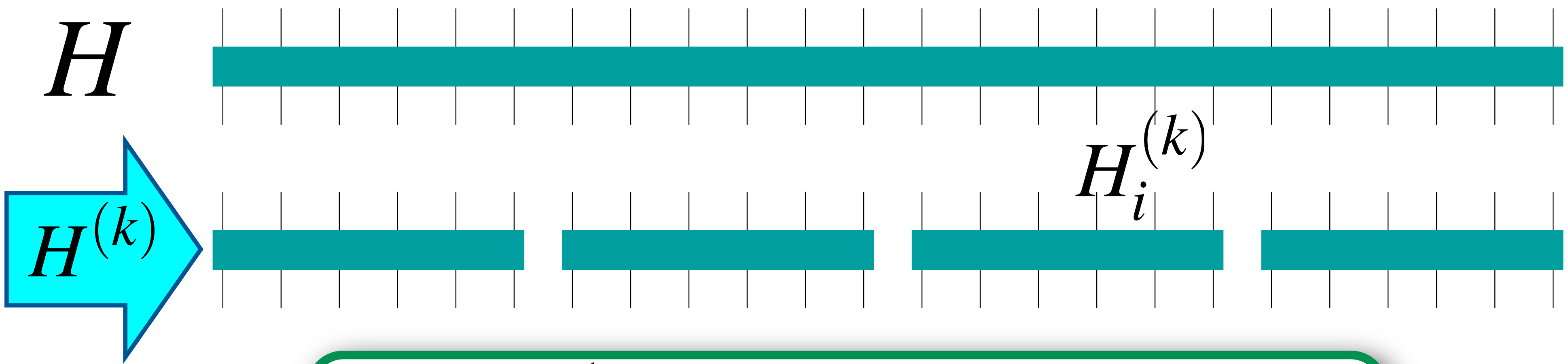
$$\lim_{N\to\infty}\left(e^{-i\frac{\omega t}{N}H^{(0)}}e^{-i\frac{\omega t}{N}H^{(1)}}\dots e^{-i\frac{\omega t}{N}H^{(p-1)}}\right)^N = e^{-i\omega t H}$$



Trotter-ization of H

QCFT

Trotter-ization of H



$$H = \sum_{k=0}^{p-1} H^{(k)}, \quad H^{(k)} = \sum_{i=-N_x}^{N_x} H_i^{(k)}$$
$$[H_i^{(k)}, H_j^{(k)}] = 0, \quad [H^{(l)}, H^{(k)}] \neq 0$$

(Ichinose and Tamura bound)

$$\left\| e^{-i\omega t H} - \left(e^{-i\frac{\omega t}{N} H^{(0)}} e^{-i\frac{\omega t}{N} H^{(1)}} \ldots e^{-i\frac{\omega t}{N} H^{(p-1)}} \right)^N \right\| \leqslant \mathcal{O}(N^{-1})$$

SIMULATING QFT

Simple scalar fields in 1 space dimension

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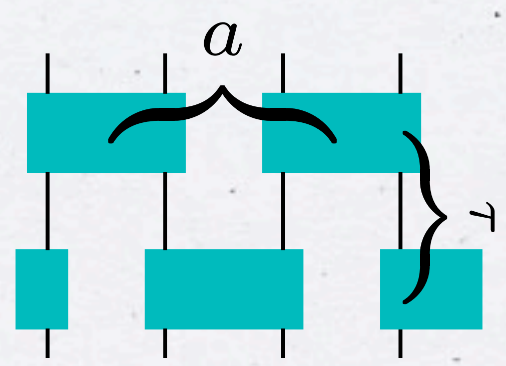
a space-granularity (minimal in principle discrimination between independent events);



τ time-granularity;

$\phi(x)$: field, operator function of space (evolving in time); we will describe it by the set of operators $\phi_n := a^{\frac{1}{2}}\phi(na)$

ϕ_n generally nonlocal operators. In QFT they satisfy (anti)commutation relations

Equal-time microcausality:

Fermion: $\psi \quad \{\psi_n, \psi_m\} = \delta_{nm}$ (Dirac)

Boson: $\varphi \quad [\varphi_n, \varphi_m] = \delta_{nm}$ (Newton-Wigner)

SIMULATING QFT

Simple scalar fields in 1 space dimension

Time evolution: $\boxed{\phi(t) = U_t^\dagger \phi(0) U_t}$

$$U_t = \exp\left(-\frac{i}{\hbar} t\hbar\omega H\right) = \exp(-2\pi i N_T H)$$

H: adimensional Hamiltonian

$$N_T = \frac{t}{T} = \frac{\omega t}{2\pi}$$

$$\boxed{i\hbar\partial_t \phi_n = [\phi_n, \hbar\omega H]}$$

SIMULATING QFT

Klein-Gordon in 1 space dimension

$$H_s = -s\frac{i}{2}\sum_n (\phi_n^{(s)\dagger}\phi_{n+1}^{(s)} - \phi_{n+1}^{(s)}{}^{\dagger}\phi_n^{(s)}) = s\frac{a}{\hbar}P,$$

$s = \pm 1$

$\phi_n := a^{\frac{1}{2}}\phi(na)$

$P = -i\hbar\int \mathrm{d}x\,\phi^\dagger(x)\partial_x\phi(x)$

$$[\phi^{(s)}(x), H_s] = [a^{-\frac{1}{2}}\phi_n^{(s)}, H_s] = -a^{-\frac{1}{2}}s\frac{i}{2}(\phi_{n+1}^{(s)} - \phi_{n-1}^{(s)}) = -isa\partial_x\phi^{(s)}(x)$$

$\omega a = c$ $\Rightarrow$ $\Box\phi = 0$

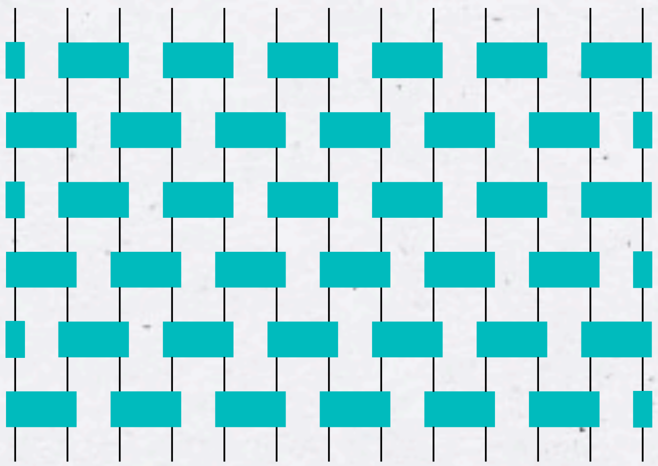
Both for Bose and Fermi fields (using: $[AB, C] = A[B, C]_\pm \mp [A, C]_\pm B$)

SIMULATING QFT

Klein-Gordon in 1 space dimension

Trotter-ization

$$U_t = e^{-i\omega t H} = \lim_{N\to\infty} U_t^{(N)}, \quad U_t^{(N)} := \left[\left(\prod_l e^{-\frac{it}{2N\tau}H_{2l-1,2l}}\right)\left(\prod_l e^{-\frac{it}{2N\tau}H_{2l,2l+1}}\right)\right]^N;$$



$$\boxed{H_{n,n+1} = \mp\frac{\pi i}{4}\left(\phi_n^{(s)\dagger}\phi_{n+1}^{(s)} - \phi_{n+1}^{(s)\dagger}\phi_n^{(s)}\right)}$$

renormalized coupling

$$v_{caus} = \frac{a}{\tau} = c \Rightarrow \frac{L}{t} = \frac{2N_x a}{2N\tau} = c \Rightarrow N_x = N$$

swapping gate $\Rightarrow$ $2\pi N_T/N = \pi$

$$\omega a = c \Rightarrow \boxed{a = \frac{cT}{2\pi}, \quad \tau = \frac{T}{2\pi}, \quad t = 2N\tau.}$$

SIMULATING QFT

Dirac in 1 space dimension

$$\boxed{i\hbar\partial_t\psi = \begin{pmatrix} ic\hbar\sigma_x\partial_x & mc^2 \\ mc^2 & -ic\hbar\sigma_x\partial_x \end{pmatrix}\psi,}$$

$$\psi(x) = \begin{pmatrix} \psi^1(x) \\ \psi^2(x) \\ \psi^3(x) \\ \psi^4(x) \end{pmatrix} := \begin{pmatrix} u(x) \\ v(x) \end{pmatrix},$$

Field equal-time commutation relations (quantization rules)

$$\{\psi^\alpha(x), \psi^{\dagger\beta}(y)\} = \delta_{\alpha\beta}\delta(x-y), \qquad \{\psi^\alpha(x), \psi^\beta(y)\} = 0.$$

Hamiltonian:

$$\hbar\omega H = \int \mathrm{d}x\, \psi^\dagger(x) \begin{pmatrix} ic\hbar\sigma_x\partial_x & mc^2 \\ mc^2 & -ic\hbar\sigma_x\partial_x \end{pmatrix}\psi(x)$$

SIMULATING QFT

Dirac in 1 space dimension

QCFT $\psi_n^\alpha = a^{\frac{1}{2}} \psi^\alpha(na)$

$$\psi_n^\alpha = \Gamma_{4n+\alpha}, \quad \Gamma_k := \left(\prod_{j=-\infty}^{k-1} \sigma_j^z\right) \sigma_k^-, \quad \{\Gamma_k, \Gamma_h\} = \delta_{kh}$$

$$i\hbar\partial_t \psi_n = [\psi_n, \hbar\omega H]$$

SIMULATING QFT

Dirac in 1 space dimension

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QCFT for $\omega a = c$

using the identity

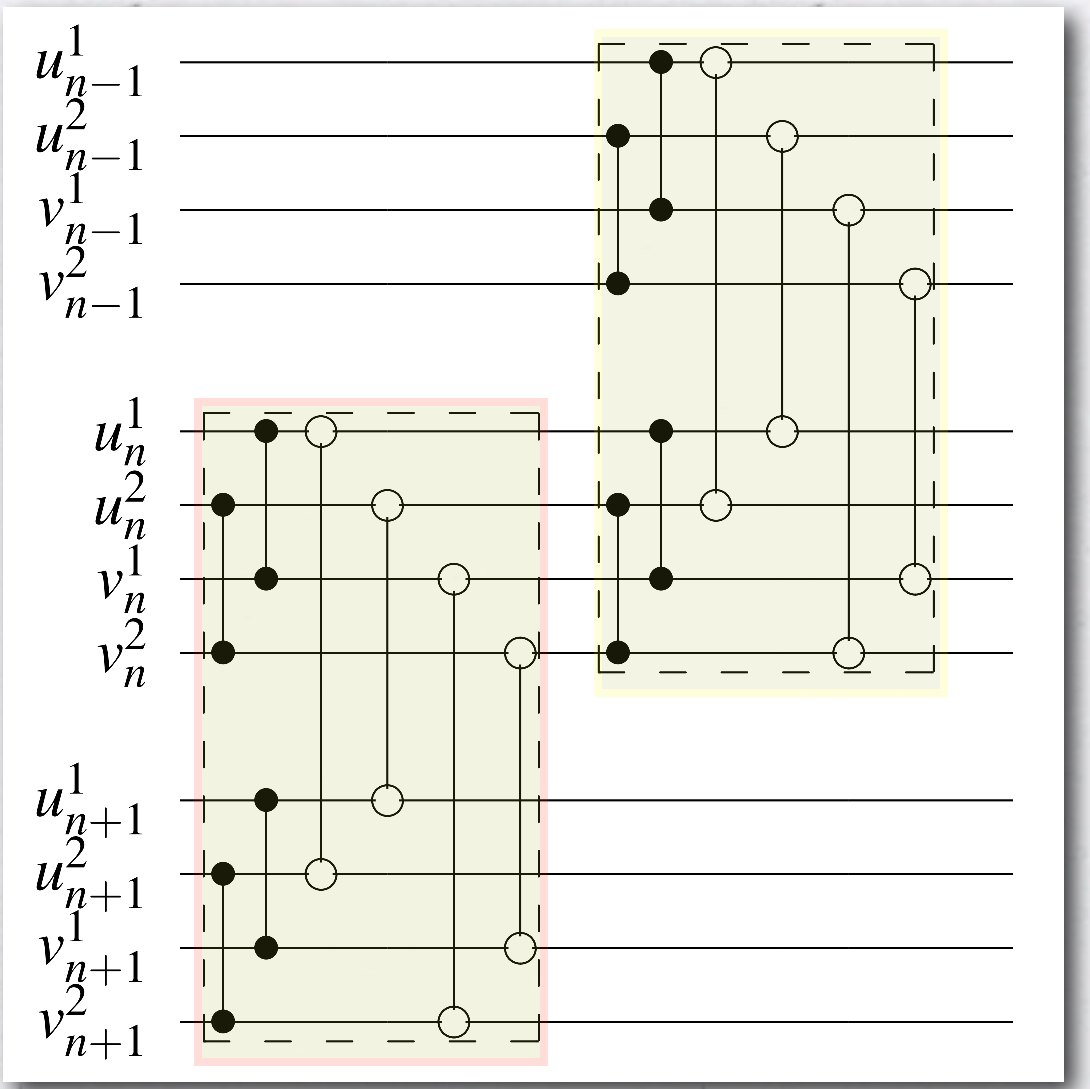
$$\left[\sum_{n\alpha} \psi_n^{\alpha\dagger} K \psi_n^{\alpha}, \psi_l^{\beta}\right] = -\sum_{n\alpha} \{\psi_n^{\alpha\dagger}, \psi_l^{\beta}\} K \psi_n^{\alpha} = -K\psi_l^{\beta}$$

$$H = \sum_{n\alpha} \psi_{n\alpha}^{\dagger} \begin{pmatrix} \frac{i}{2}\sigma_x(\delta_+ - \delta_-) & \frac{a}{\lambda} I \\ \frac{a}{\lambda} I & -\frac{i}{2}\sigma_x(\delta_+ - \delta_-) \end{pmatrix} \psi_{n\alpha}$$

$$\lambda := \frac{\hbar}{mc} = 3.86159 * 10^{-13}$$

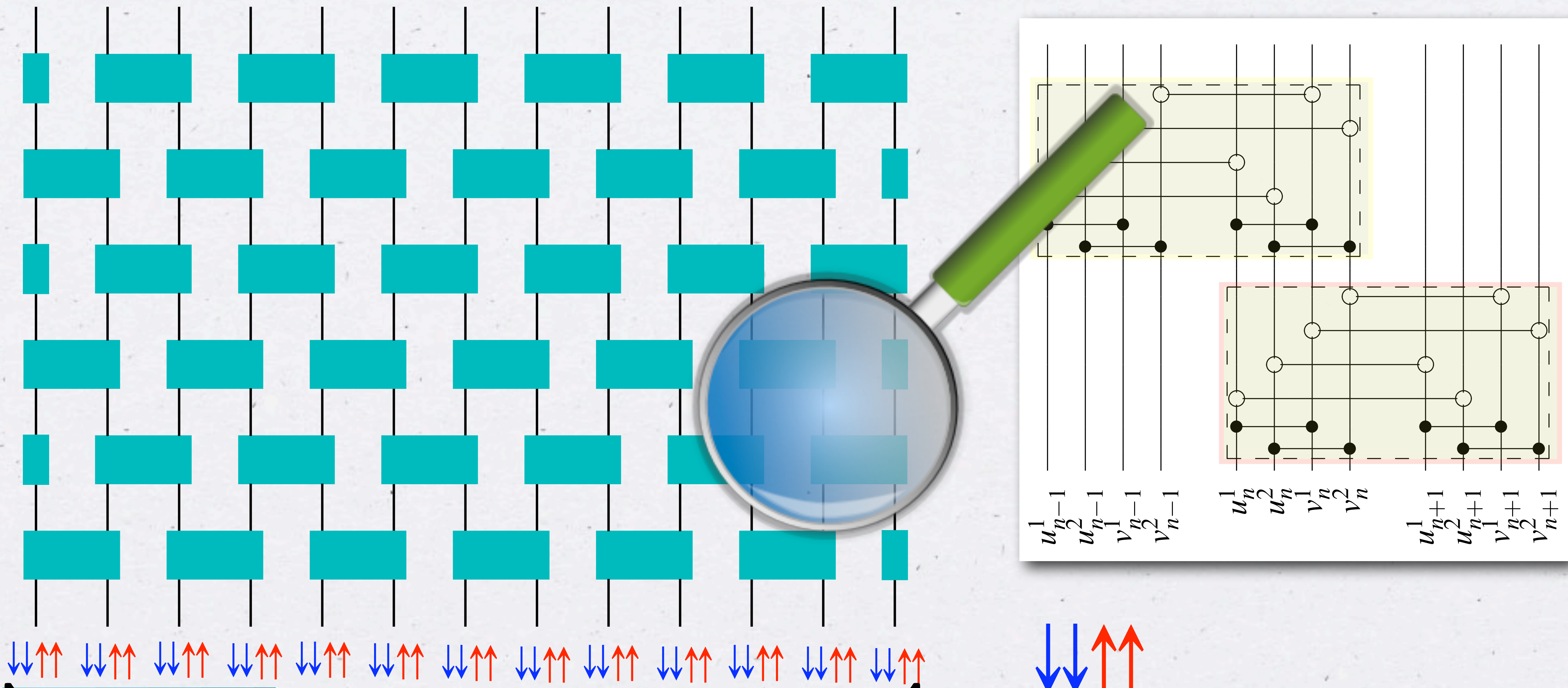
Compton wavelength

∂_x makes sense above the scale of λ



SIMULATING QFT

Dirac in 1 space dimension: **the vacuum**



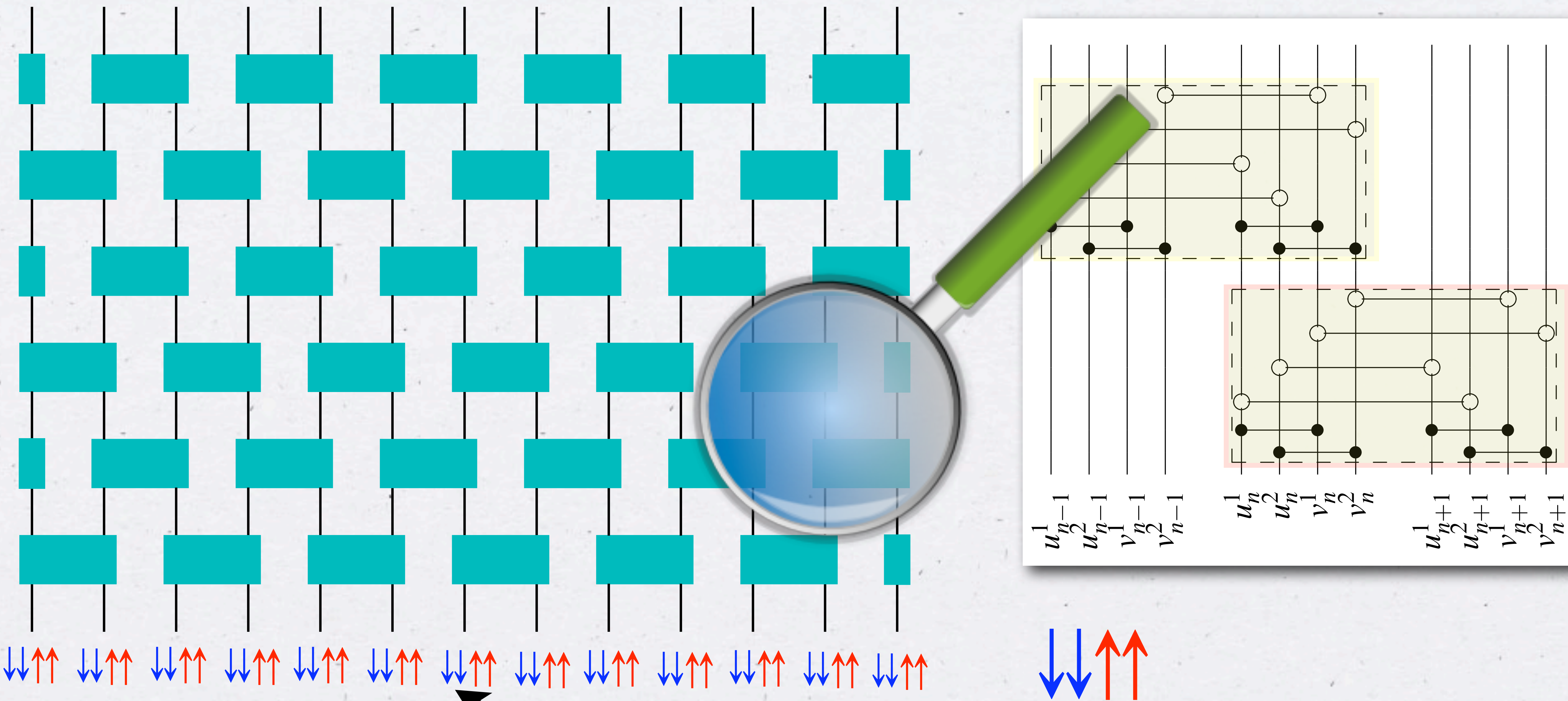
Dirac filled sea

antiparticle

particle spin-up and spin down

SIMULATING QFT

Dirac in 1 space dimension



"particle" creation

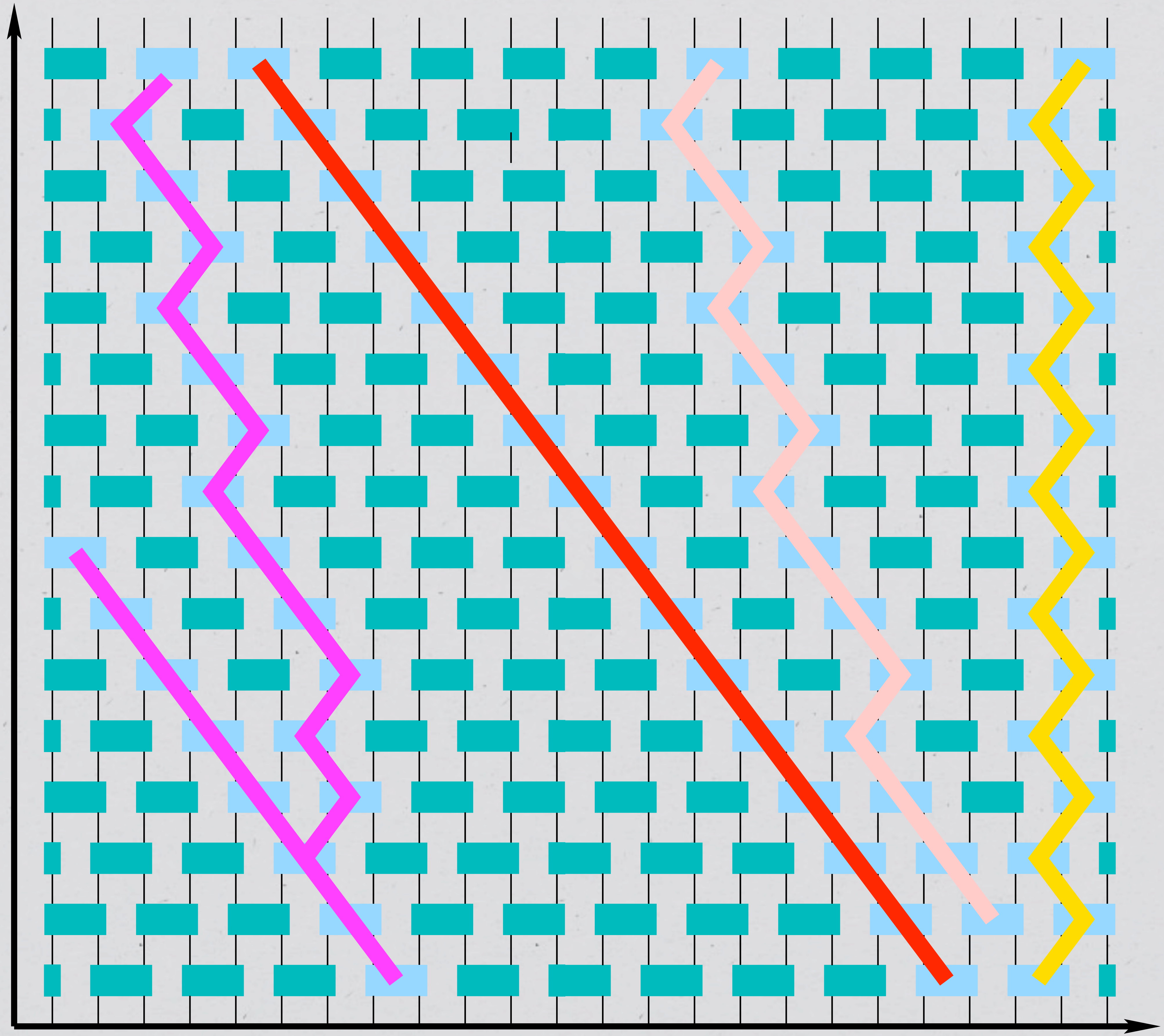
antiparticle

particle spin-up and spin down

$$|\psi_n^1\rangle := \psi_n^{1\dagger}|0\rangle$$

$$\boxed{\psi_n^{1\dagger} = \left(\prod_{j=-\infty}^{4n} \sigma_j^z\right)\sigma^+_{4n+1}}$$

Zitterbewegung



SIMULATING QFT

Dirac in 1 space dimension

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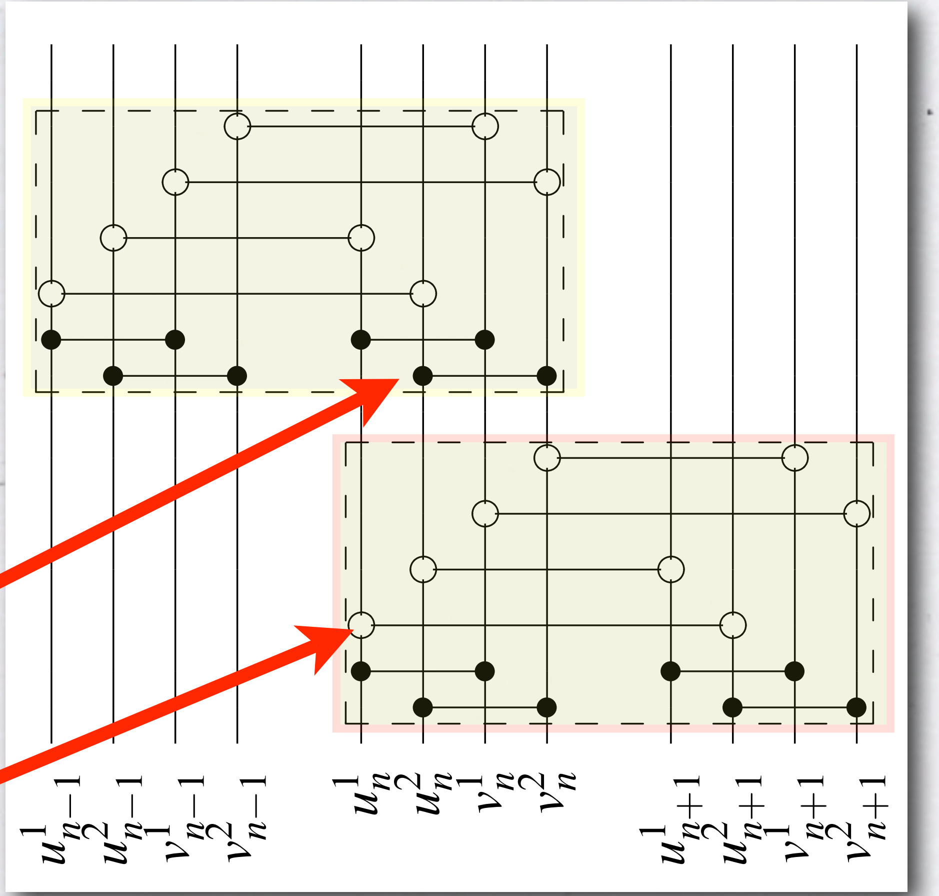
$$\psi_n^\alpha = \Gamma_{4n+\alpha}, \quad \Gamma_k := \left(\prod_{j=-\infty}^{k-1} \sigma_j^z \right) \sigma_k^-$$

$$\boxed{H = \sum_{n\alpha} \psi_{n\alpha}^\dagger \begin{pmatrix} \frac{i}{2}\sigma_x(\delta_+ - \delta_-) & \frac{a}{\lambda} I \\ \frac{a}{\lambda} I & -\frac{i}{2}\sigma_x(\delta_+ - \delta_-) \end{pmatrix} \psi_{n\alpha}}$$



$$\sigma_k^+ \sigma_{k+1}^z \sigma_{k+2}^z \cdots \sigma_{k+l-1}^z \sigma_{k+l}^-$$

$$\sigma_h^- \sigma_{h+1}^z \sigma_{h+2}^z \cdots \sigma_{h+l-1}^z \sigma_{h+l}^+$$

$$|\psi_n^1\rangle := \psi_n^{1\dagger}|0\rangle \quad |0\rangle := \downarrow\downarrow\uparrow\uparrow\ \downarrow\downarrow\uparrow\uparrow\ \downarrow\downarrow\uparrow\uparrow\ \downarrow\downarrow\uparrow\uparrow\ \downarrow\downarrow\uparrow\uparrow\ \downarrow\downarrow\uparrow\uparrow\ \downarrow\downarrow\uparrow\uparrow\ \downarrow\downarrow\uparrow\uparrow\ \downarrow\downarrow\uparrow\uparrow\ \downarrow\downarrow\uparrow\uparrow\ \downarrow\downarrow\uparrow\uparrow\ \downarrow\downarrow\uparrow\uparrow\ \downarrow\downarrow\uparrow\uparrow$$

ONLY QUBITS! NO MORE FIELDS!
NO MORE QUANTIZATION RULES!

QCFT OF DIRAC

1 space dimension

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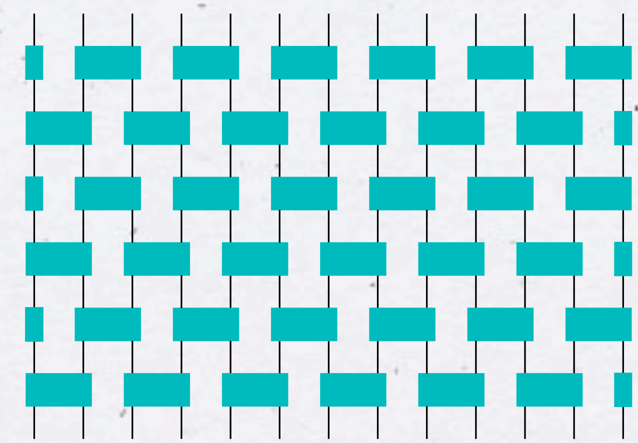
$$\zeta^1=\sigma^-,\zeta^2=\sigma_1\sigma_2^-,\zeta^3=\sigma_1^z\sigma_2^z\sigma_3^-,\zeta^4=\sigma_1^z\sigma_2^z\sigma_3^z\sigma_4^-,\zeta^5=\sigma_1^z\sigma_2^z\sigma_3^z\sigma_4^z$$

$$\left(\zeta^5\right)^2=I,\qquad \{\zeta^i,\zeta^j\}=0,\quad \{\zeta^i,\zeta^{j\dagger}\}=\delta_{ij},\qquad i,j=1,\dots,4$$



$$H_{n,n+1}=\frac{\pi}{2}\begin{pmatrix}\zeta_n^1\\ \zeta_n^2\\ \zeta_n^3\\ \zeta_n^4\end{pmatrix}^{\dagger}\begin{pmatrix}0 & i\Gamma\overleftrightarrow{\Delta} & \gamma & 0\\ i\Gamma\overleftrightarrow{\Delta} & 0 & 0 & \gamma\\ \gamma & 0 & 0 & -i\Gamma\overleftrightarrow{\Delta}\\ 0 & \gamma & -i\Gamma\overleftrightarrow{\Delta} & 0\end{pmatrix}\begin{pmatrix}\zeta_n^1\\ \zeta_n^2\\ \zeta_n^3\\ \zeta_n^4\end{pmatrix}$$

$$\overleftrightarrow{\Delta}:=\tfrac{1}{2}(\overrightarrow{\delta}_+-\overleftarrow{\delta}_+)\qquad |0\rangle:=\Big|\cdots\underbrace{\downarrow\downarrow\uparrow\uparrow}_{n-1}\underbrace{\downarrow\downarrow\uparrow\uparrow}_{n}\underbrace{\downarrow\downarrow\uparrow\uparrow}_{n+1}\cdots\Big\rangle$$

QCFT OF DIRAC

Recovering QFT from QCFT

$$H_{n,n+1} = \frac{\pi}{2} \begin{pmatrix} \zeta_n^1 \\ \zeta_n^2 \\ \zeta_n^3 \\ \zeta_n^4 \end{pmatrix}^\dagger \begin{pmatrix} 0 & i\Gamma\overleftrightarrow{\Delta} & \gamma & 0 \\ i\Gamma\overleftrightarrow{\Delta} & 0 & 0 & \gamma \\ \gamma & 0 & 0 & -i\Gamma\overleftrightarrow{\Delta} \\ 0 & \gamma & -i\Gamma\overleftrightarrow{\Delta} & 0 \end{pmatrix} \begin{pmatrix} \zeta_n^1 \\ \zeta_n^2 \\ \zeta_n^3 \\ \zeta_n^4 \end{pmatrix}$$

Recovering QFT for $t \gg \tau$ and $x \gg \lambda$

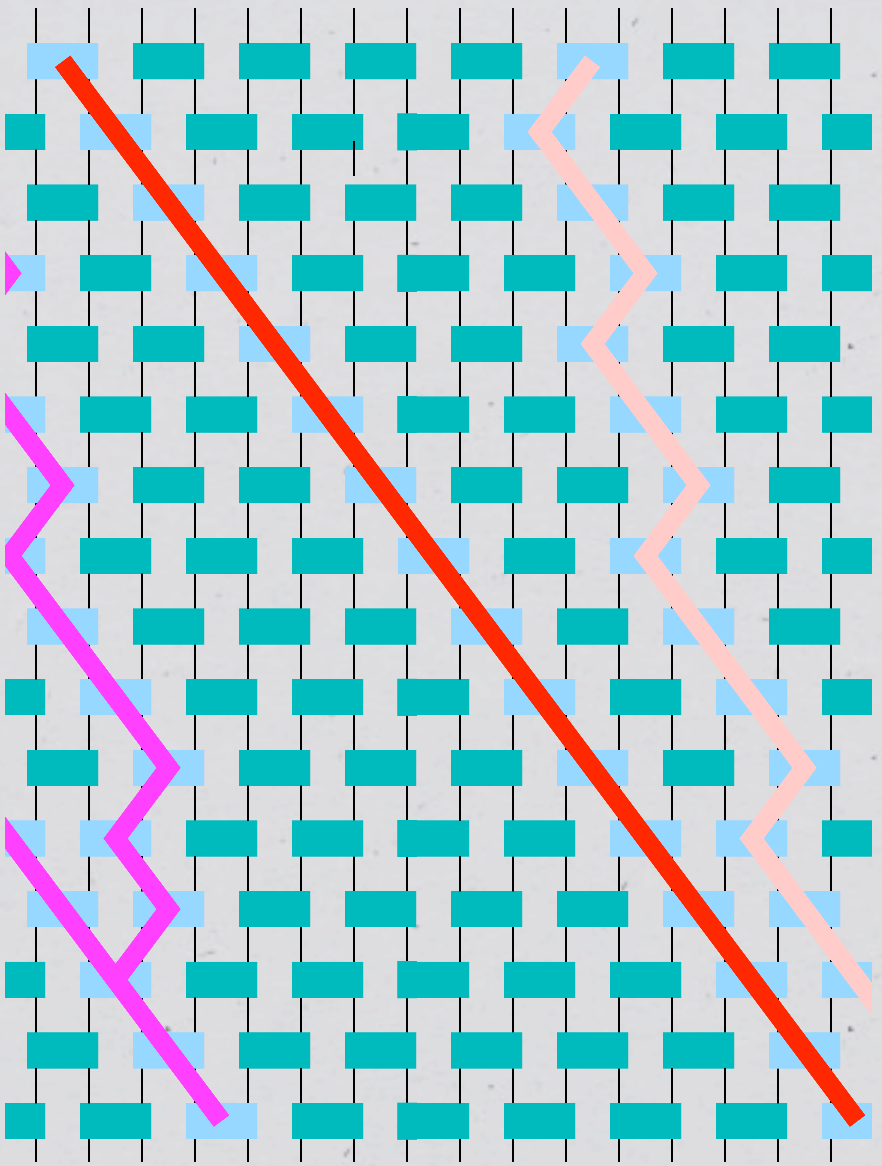
$$\lambda = \frac{\Gamma}{\gamma} a$$

$$\boxed{\psi_n^\alpha = \left(\prod_{j=-\infty}^{n-1} \zeta_j^5 \right) \zeta_n^\alpha} \quad \boxed{c = \frac{a}{\tau}} \quad \boxed{\hbar = \Gamma\tau} \quad \boxed{mc^2 = \gamma}$$

$$\frac{1}{2a}(\delta_+ - \delta_-) = \partial_x, \qquad \frac{1}{2}(\delta_+ + \delta_-) = 1. \quad \mathcal{O}(a^2)$$

QCFT OF DIRAC

- The Zitterbewegung provides the new intuitive picture.
- The new "particles" move at the speed of light: the mass is the coupling with the antiparticle, and the interaction produces the "slow-down".
- The field description gives a "classical" description in terms of harmonic oscillation with bilinear Hamiltonian
- quantization rules "emergent"
- no causality leakage nor localization problems



SIMULATING QFT

Dirac in **3** space dimensions?

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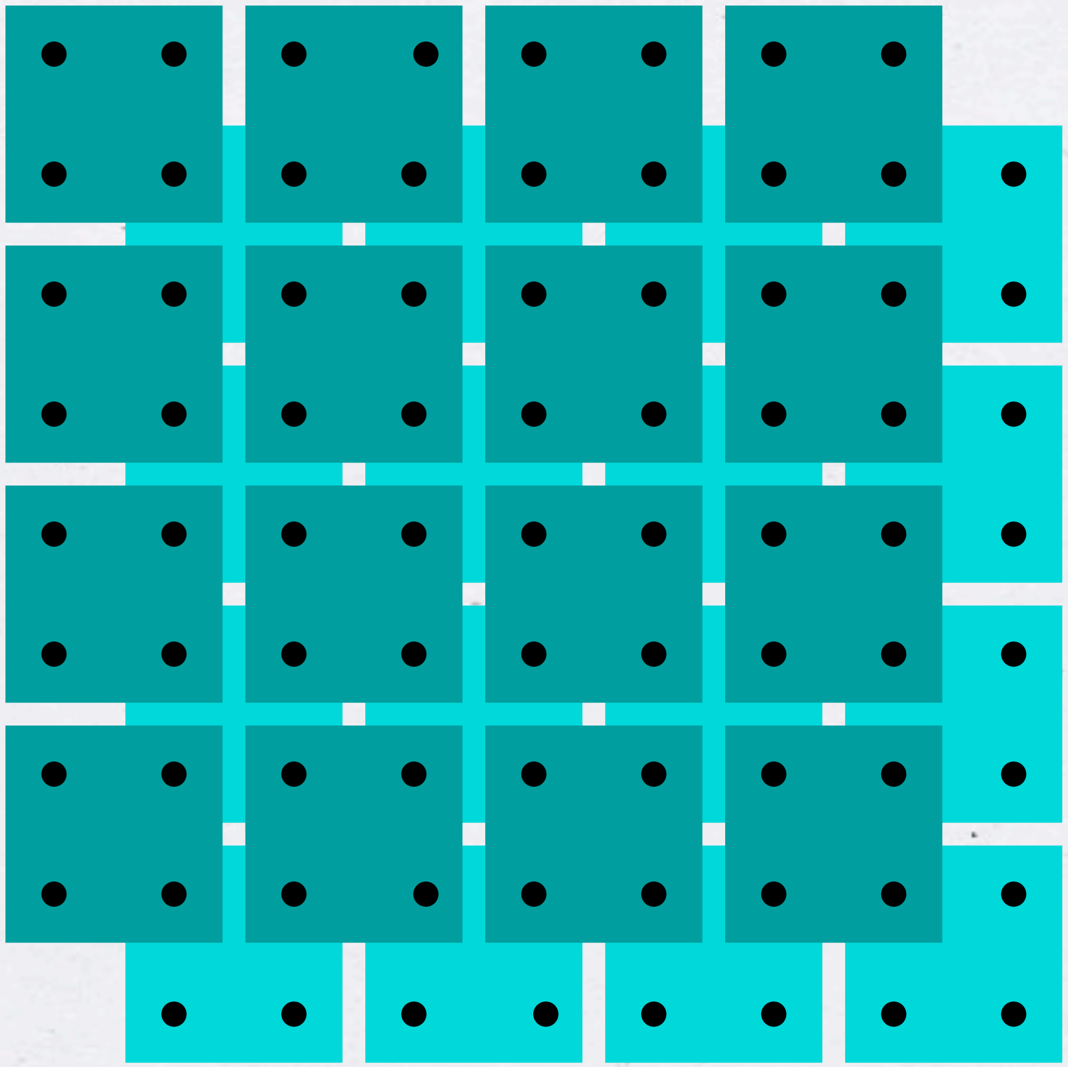

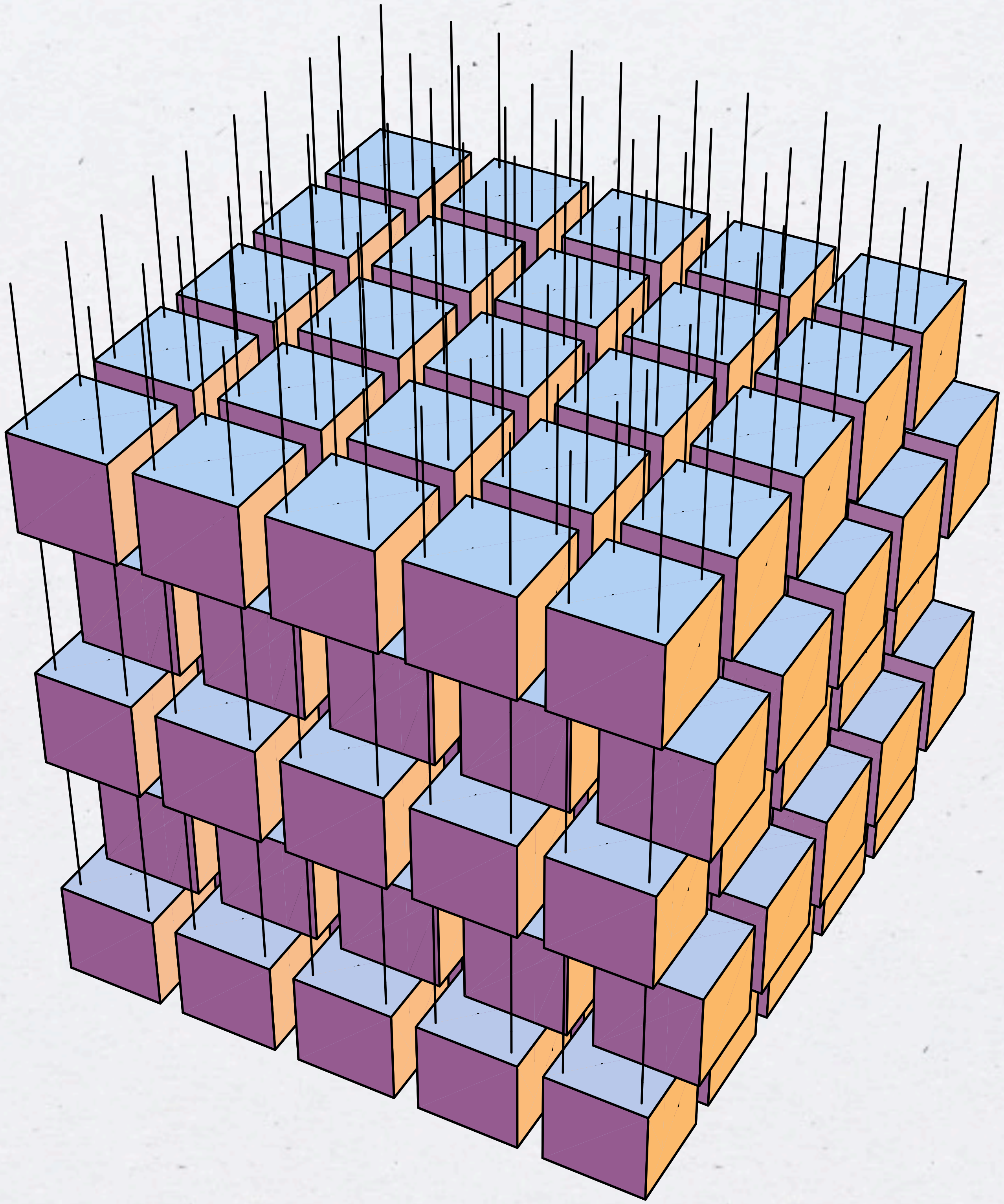

SIMULATING QFT$_1$

1ST QUANTIZATION BY QCFT$_1$

$|\phi(x)\rangle := \phi^\dagger(x)|0\rangle$ single particle at position x

$\Rightarrow$ $|\phi_n\rangle := \phi_n^\dagger|0\rangle$ qubit↑at n (or 1 boson at n)

$$i\hbar\partial_t|\phi_n\rangle = [\phi_n^\dagger, \hbar\omega H]|0\rangle = -\hbar\omega H|\phi_n\rangle$$

$$\Rightarrow \quad i\hbar\partial_t\langle\phi_n|\Phi\rangle = \hbar\omega\langle\phi_n|H|\Phi\rangle = \hbar\omega(\mathbb{H}\Phi)_n$$

$$\boxed{\Phi = \begin{pmatrix} \dots \\ \Phi_n \\ \Phi_{n+1} \\ \dots \end{pmatrix}, \quad \Phi_n = \langle\phi_n|\Phi\rangle, \qquad \mathbb{H} = \begin{pmatrix} \dots & \dots & \dots & \dots \\ \dots & \langle\phi_n|H|\phi_m\rangle & \langle\phi_n|H|\phi_{m+1}\rangle & \dots \\ \dots & \langle\phi_{n+1}|H|\phi_m\rangle & \langle\phi_{n+1}|H|\phi_{m+1}\rangle & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix}}$$

$$\langle\phi_i|\phi_n^\dagger\phi_m|\phi_j\rangle = \langle 0|\phi_i\phi_n^\dagger\phi_m\phi_j^\dagger|0\rangle = \delta_{in}\langle 0|\phi_m\phi_j^\dagger|0\rangle = \delta_{in}\delta_{jm} := (e_{nm})_{ij}$$

SIMULATING QFT 1

Dirac Particle

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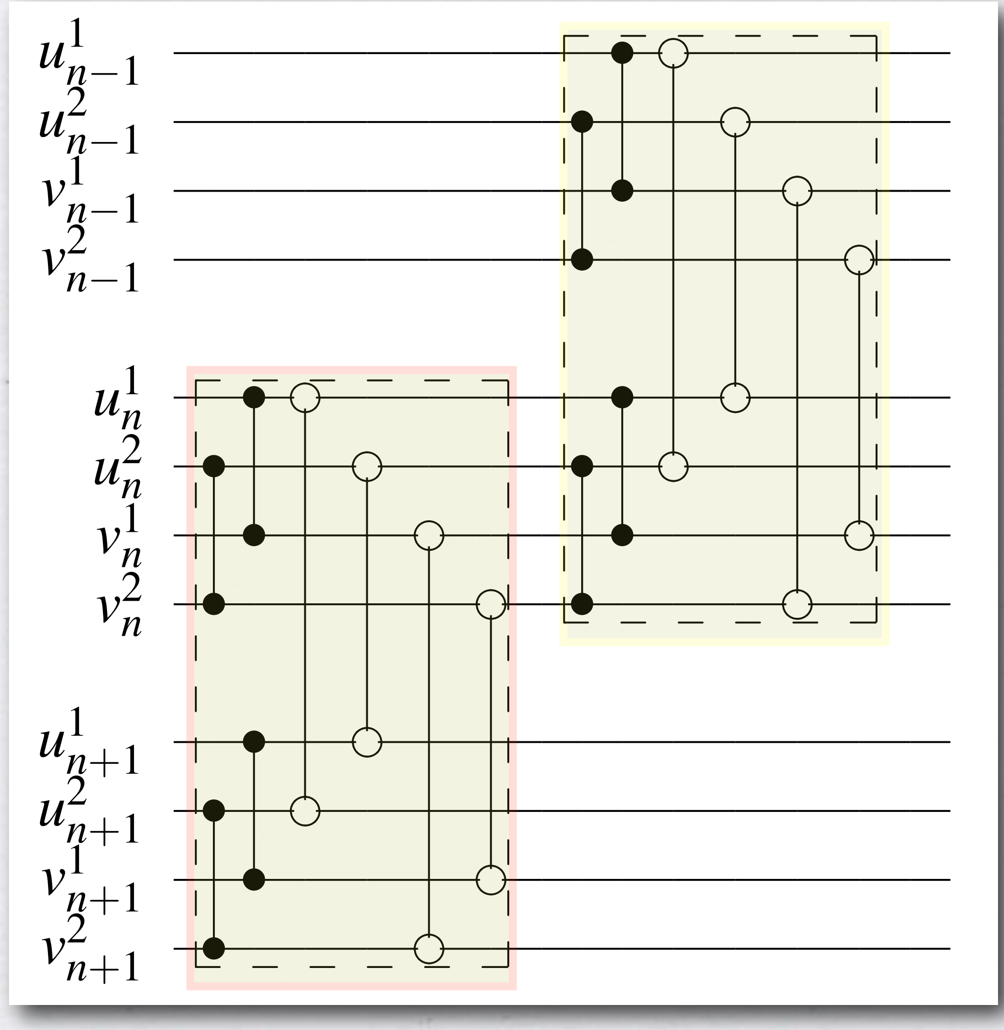
$$\boxed{i\hbar\partial_t\psi = \begin{pmatrix} ic\hbar\sigma_x\partial_x & mc^2 \\ mc^2 & -ic\hbar\sigma_x\partial_x \end{pmatrix}\psi}$$

$$\psi := \begin{pmatrix} \cdots \\ \psi_n \\ \psi_{n+1} \\ \cdots \end{pmatrix}, \qquad \psi_n = \begin{pmatrix} u_n \\ v_n \end{pmatrix} = \begin{pmatrix} u_n^1 \\ u_n^2 \\ v_n^1 \\ v_n^2 \end{pmatrix}$$

$$H = \begin{pmatrix} \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & 0 & 0 & -\frac{i}{2}\sigma_x & 0 & 0 & 0 & \cdots \\ \cdots & 0 & 0 & \frac{a}{\lambda}I & \frac{i}{2}\sigma_x & 0 & 0 & \cdots \\ \cdots & \frac{i}{2}\sigma_x & \frac{a}{\lambda}I & 0 & 0 & -\frac{i}{2}\sigma_x & 0 & \cdots \\ \cdots & 0 & -\frac{i}{2}\sigma_x & 0 & 0 & \frac{a}{\lambda}I & \frac{i}{2}\sigma_x & \cdots \\ \cdots & 0 & 0 & \frac{i}{2}\sigma_x & \frac{a}{\lambda}I & 0 & 0 & \cdots \\ \cdots & 0 & 0 & 0 & -\frac{i}{2}\sigma_x & \frac{a}{\lambda}I & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{pmatrix}$$



SIMULATING QFT$_1$

Schrödinger equation

$$\partial_t \phi = i \frac{\hbar}{2m} \partial_x^2 \phi$$

$$\omega = \frac{\hbar}{2ma^2}$$

$$H = \sum_j e_{j+1,j} - 2e_{j,j} + e_{j,j+1} = \begin{pmatrix} \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ \dots & 1 & -2 & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \\ \dots & \dots & \dots & \dots & -2 & 1 & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & 1 & -2 & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & -2 & 1 & \dots \\ \dots & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \end{pmatrix}$$

$$H = H^{(0)} + H^{(1)}, \quad H^{(0)} = \sum_j H_{2j,2j+1},\ H^{(1)} = \sum_j H_{2j+1,2j+2}$$

$$H_{j,j+1} = \sum_j e_{j+1,j} - e_{j,j} - e_{j+1,j+1} + e_{j,j+1}$$

SIMULATING QFT$_1$

Schrödinger equation

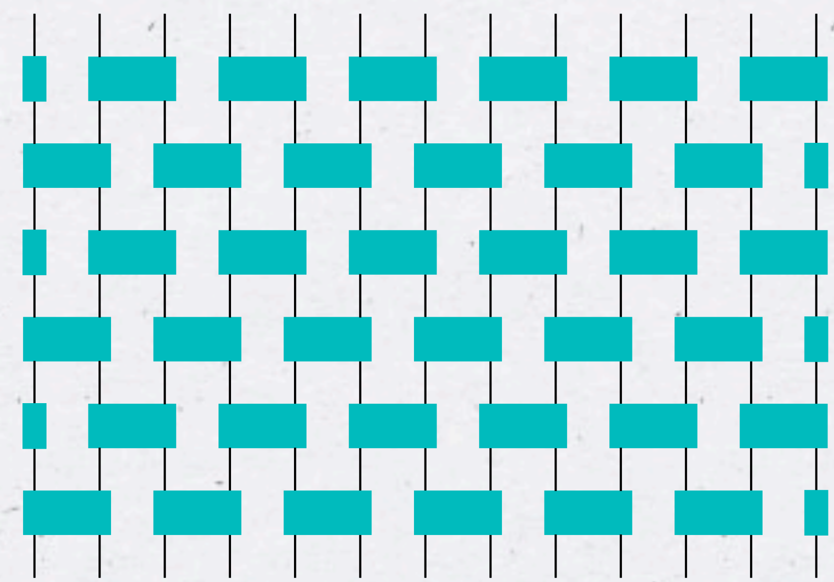
"**Trotterize**" the Hamiltonian

$$H = H^{(0)} + H^{(1)}, \quad H^{(0)} = \sum_j H_{2j,2j+1},\ H^{(1)} = \sum_j H_{2j+1,2j+2}$$

$$H_{j,j+1} = \sum_j e_{j+1,j} - e_{j,j} - e_{j+1,j+1} + e_{j,j+1}$$

By taking the maximal causal speed equal to c namely $a \propto N^{-1}$ one obtains:

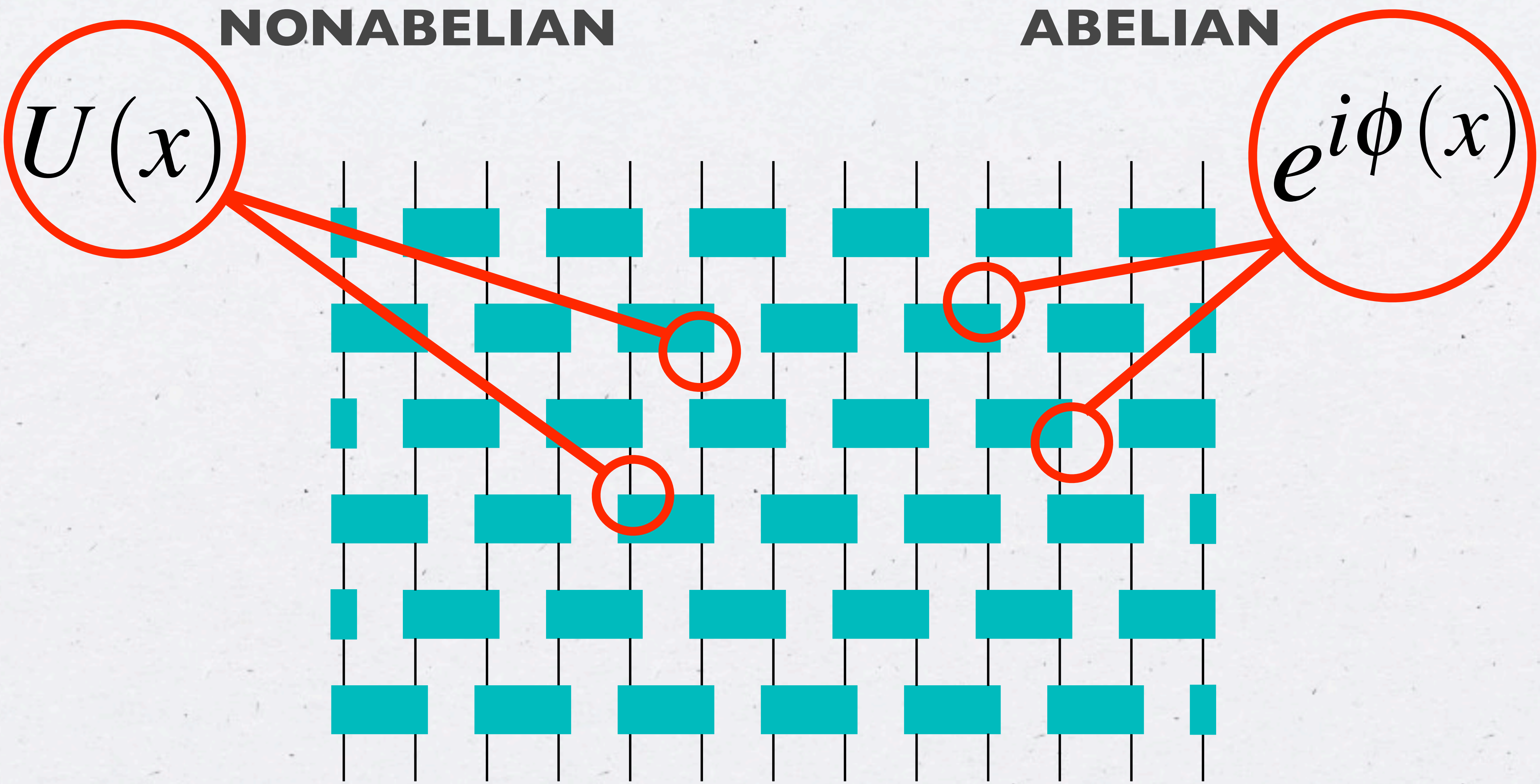
$$\boxed{\omega = \frac{\hbar}{2ma^2} \propto N^2}$$



The Schrödinger equation is not Lorentz invariant!

QCFT

GAUGE INVARIANCE



SIMULATING QFT

GAUGE INVARIANCE

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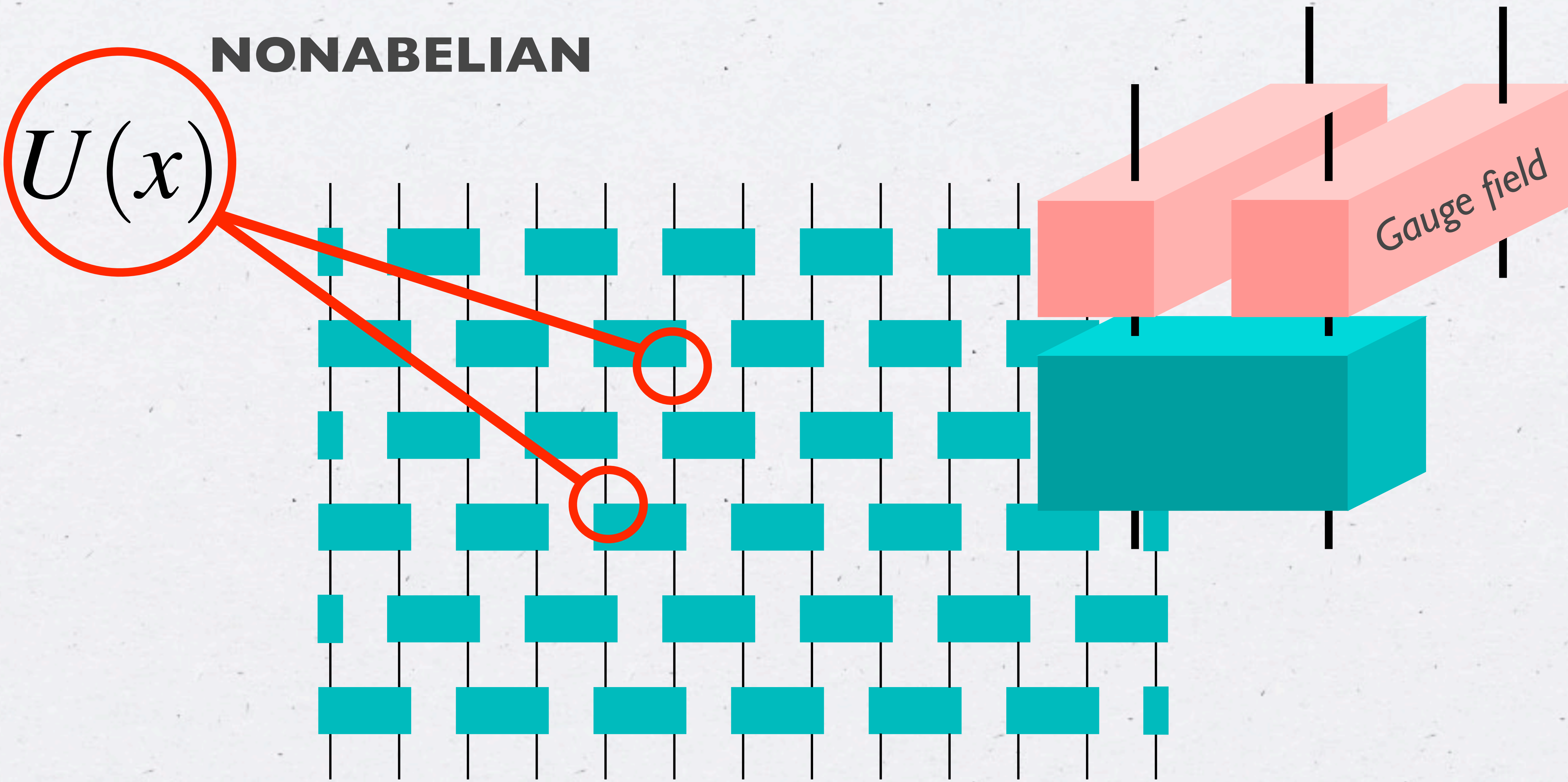
NONABELIAN



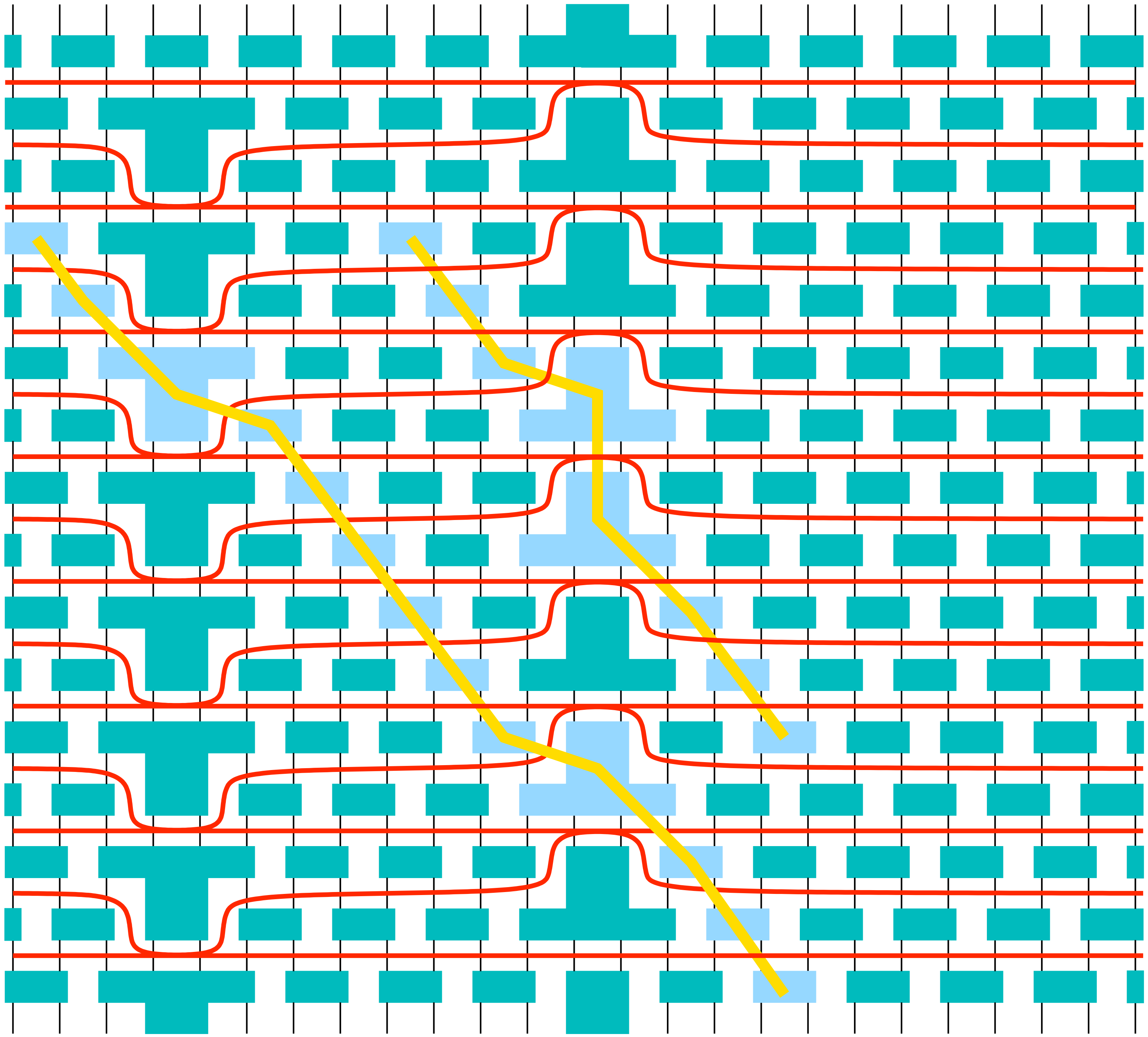
Natively nonabelian Gauge theory!
and on ... foliation !!!

Good for Gravity!

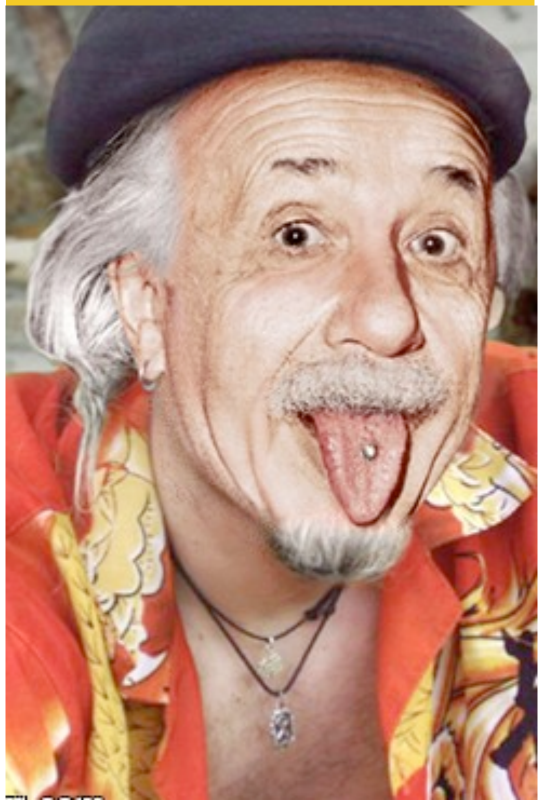
PLAY GOD WITH QCFT

or else: Einstein demystified

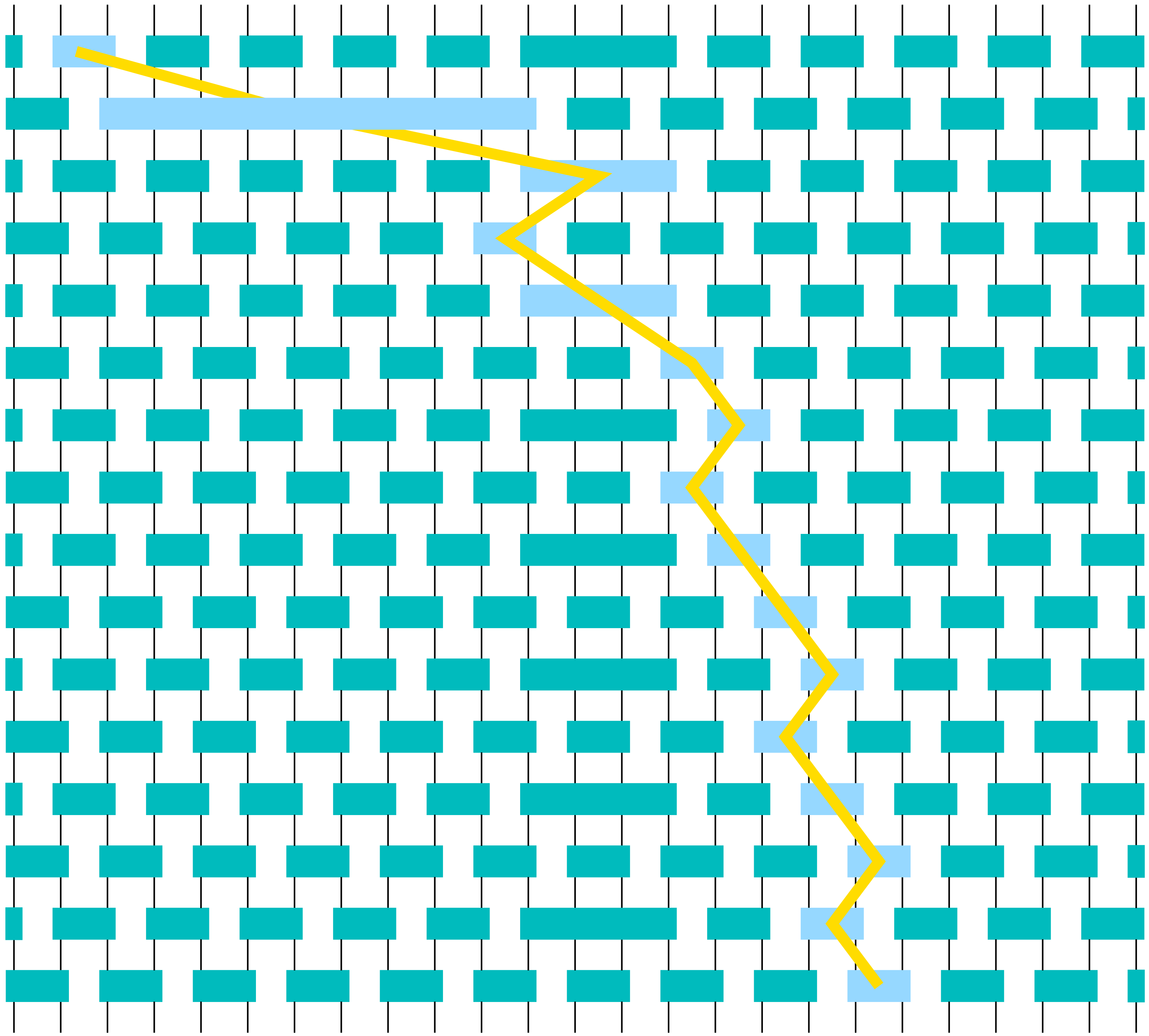
GR from QT?



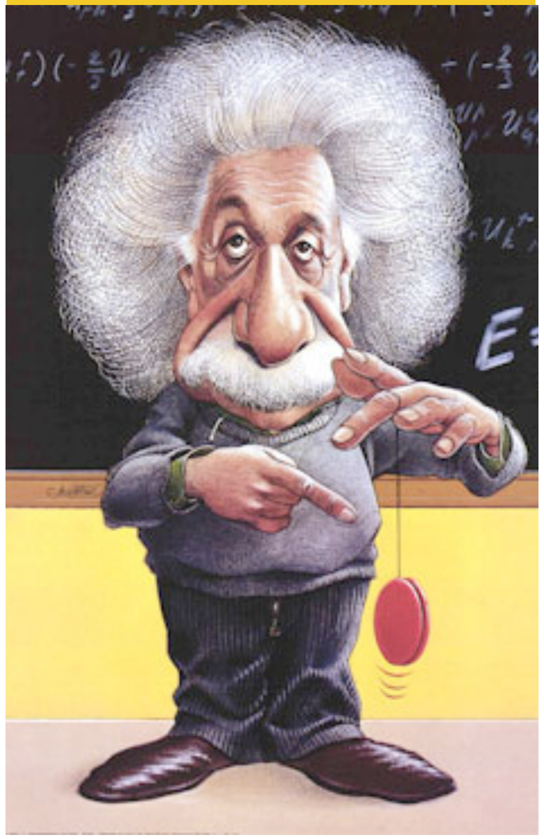
positive and negative masses



GR from QT?

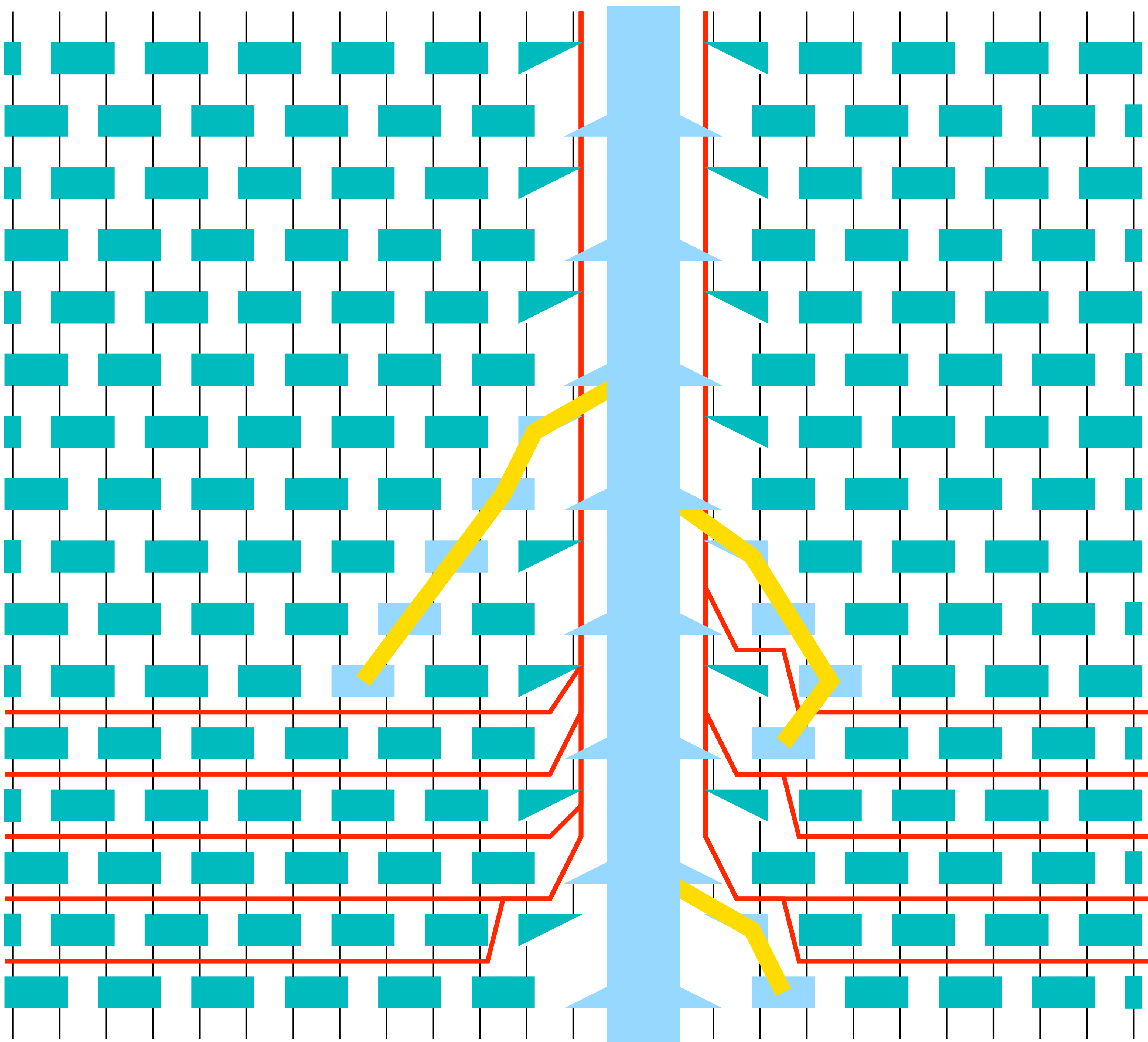


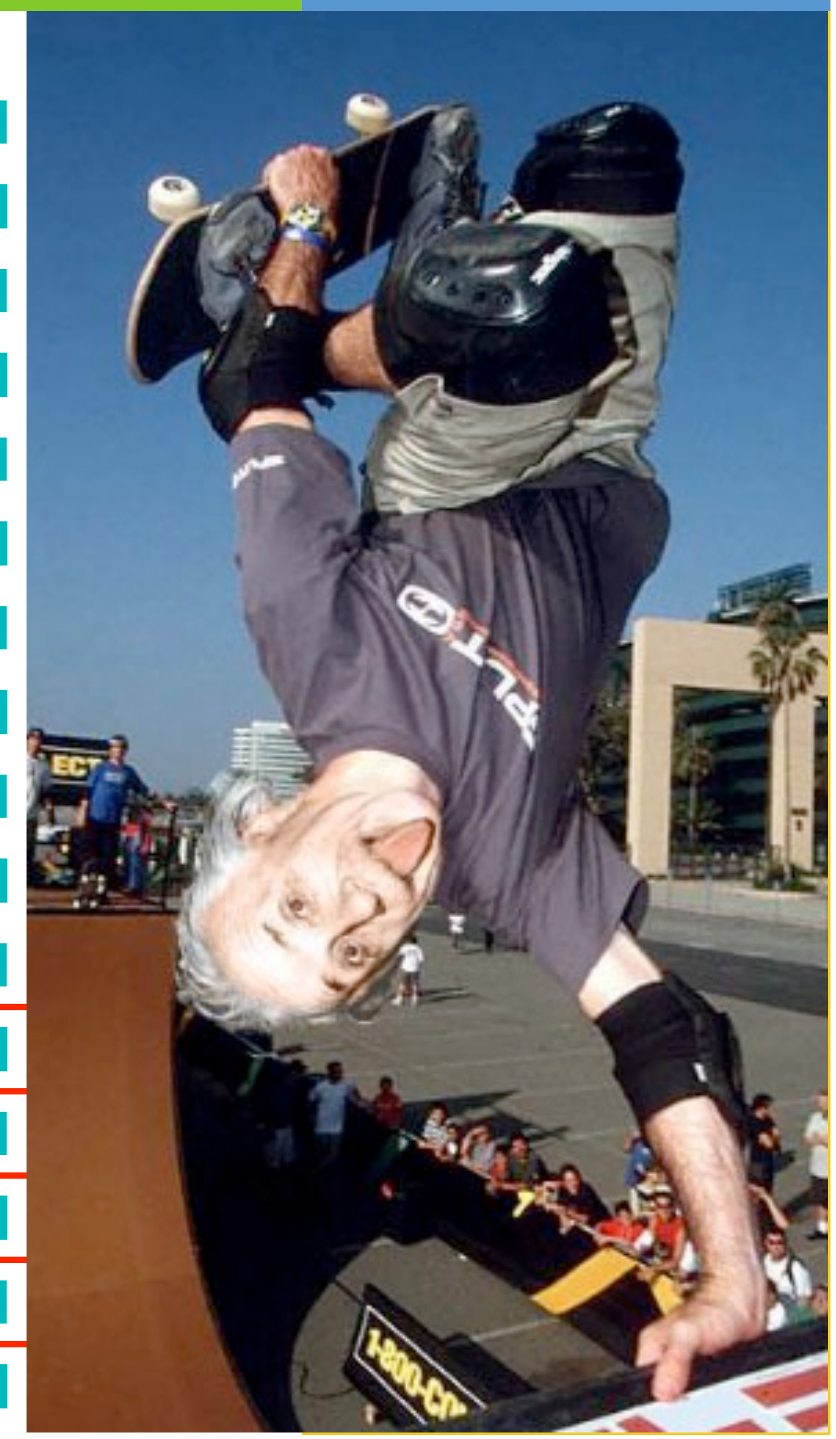
a worm hole!



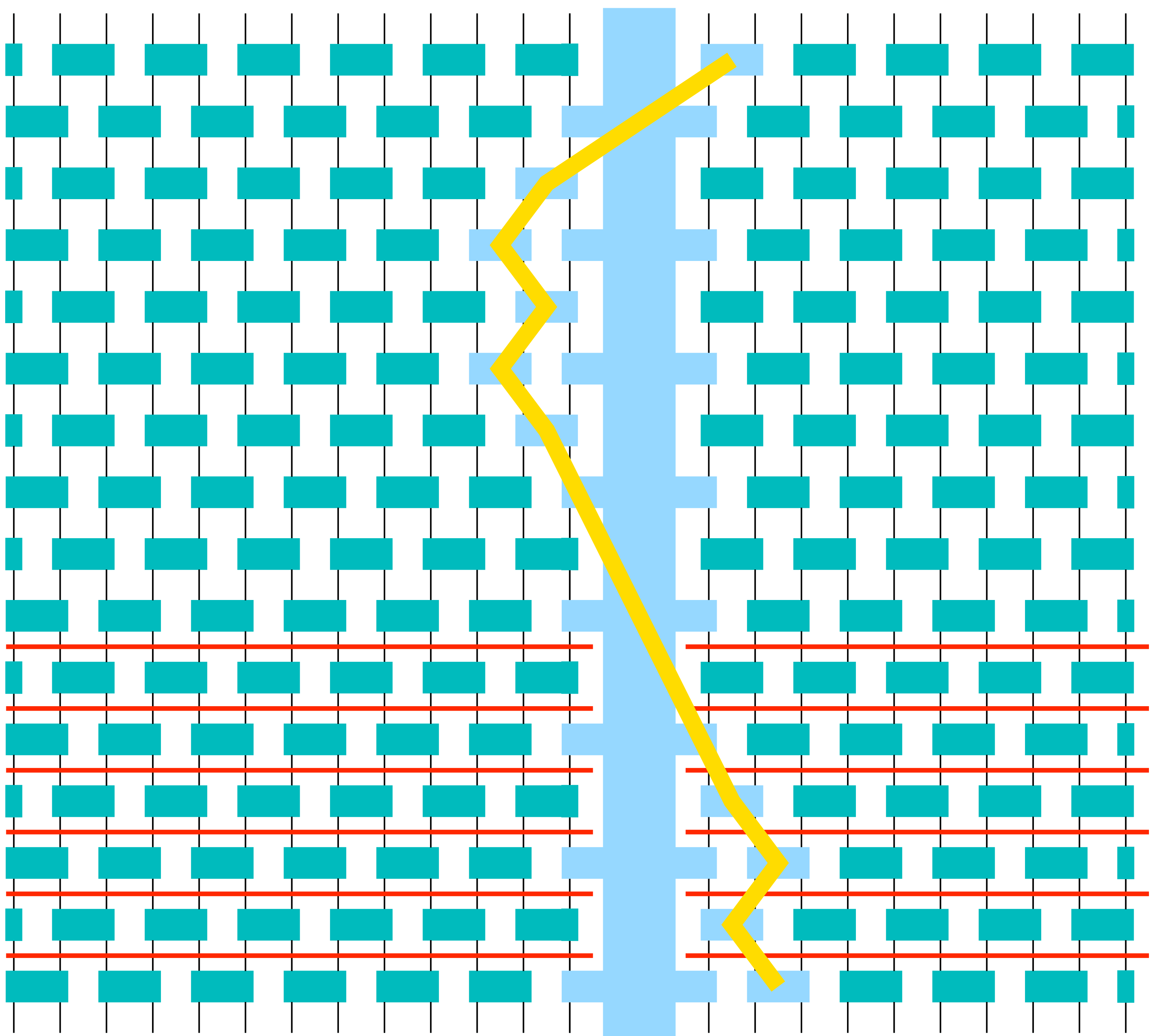
GR from QT?

a black hole!



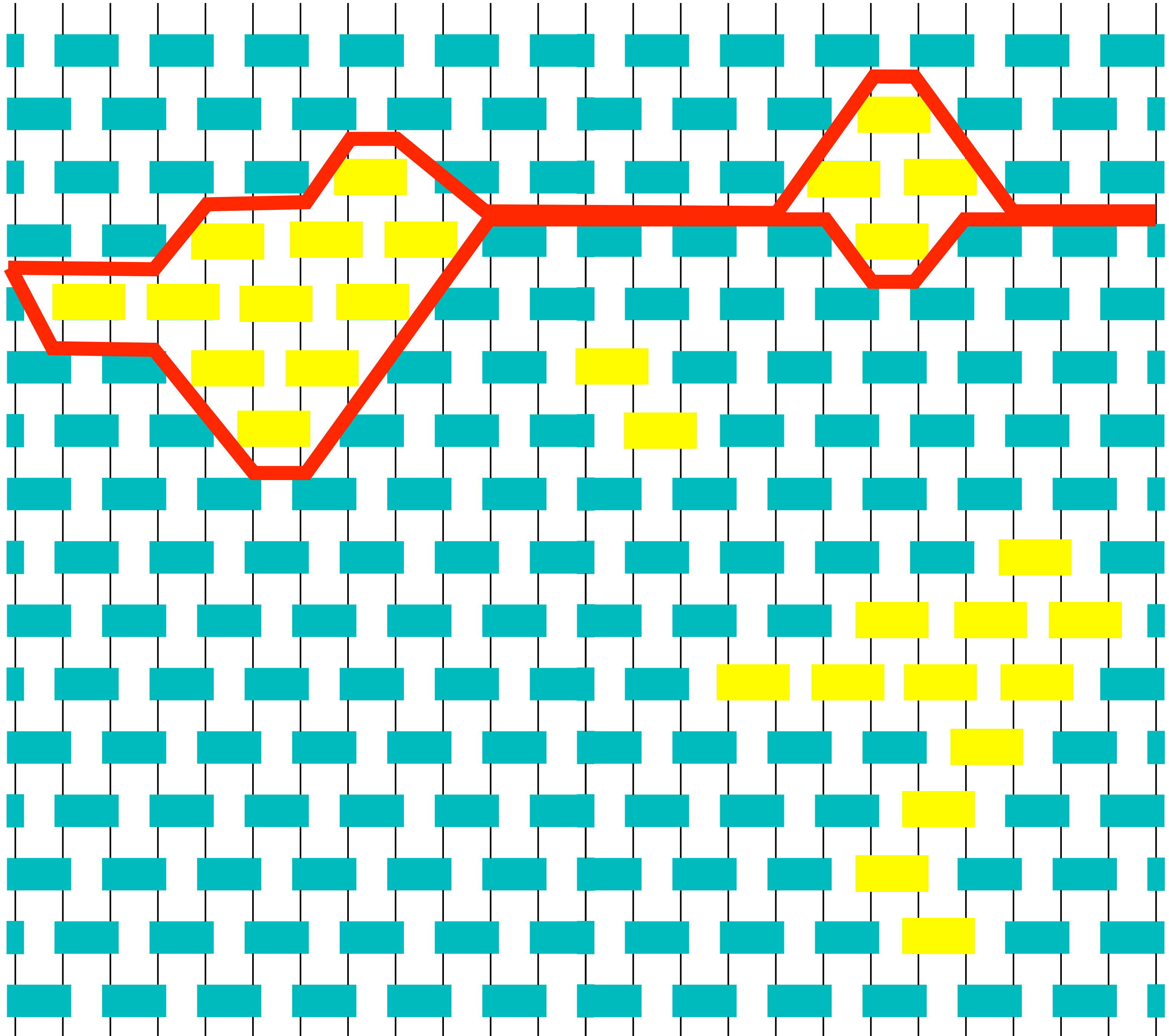


GR from QT?

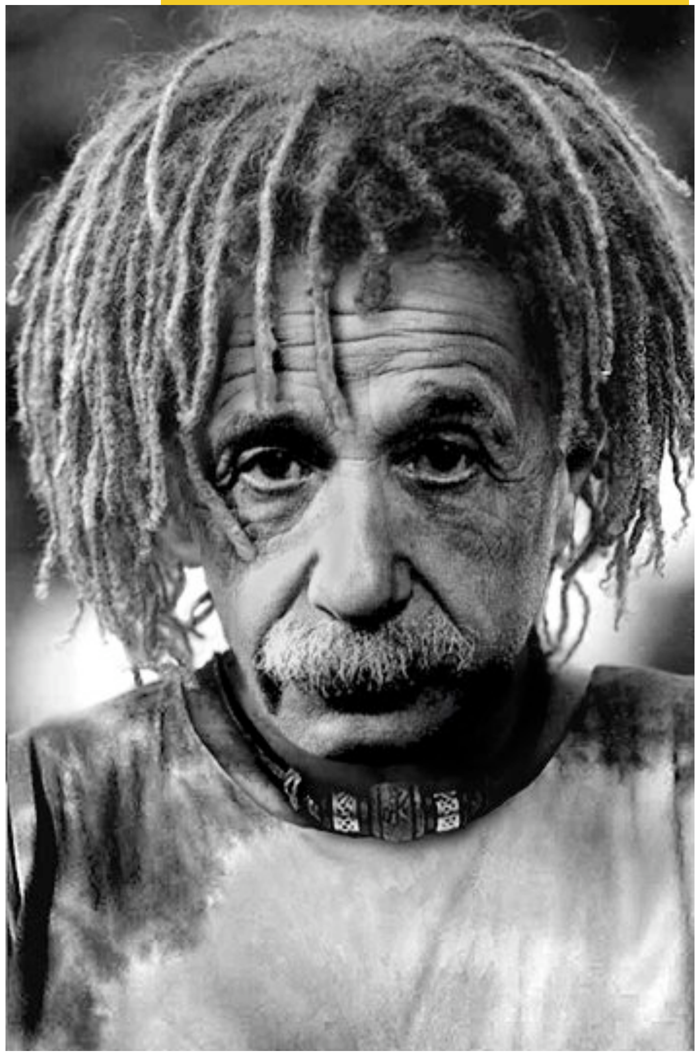


a time tunnel!

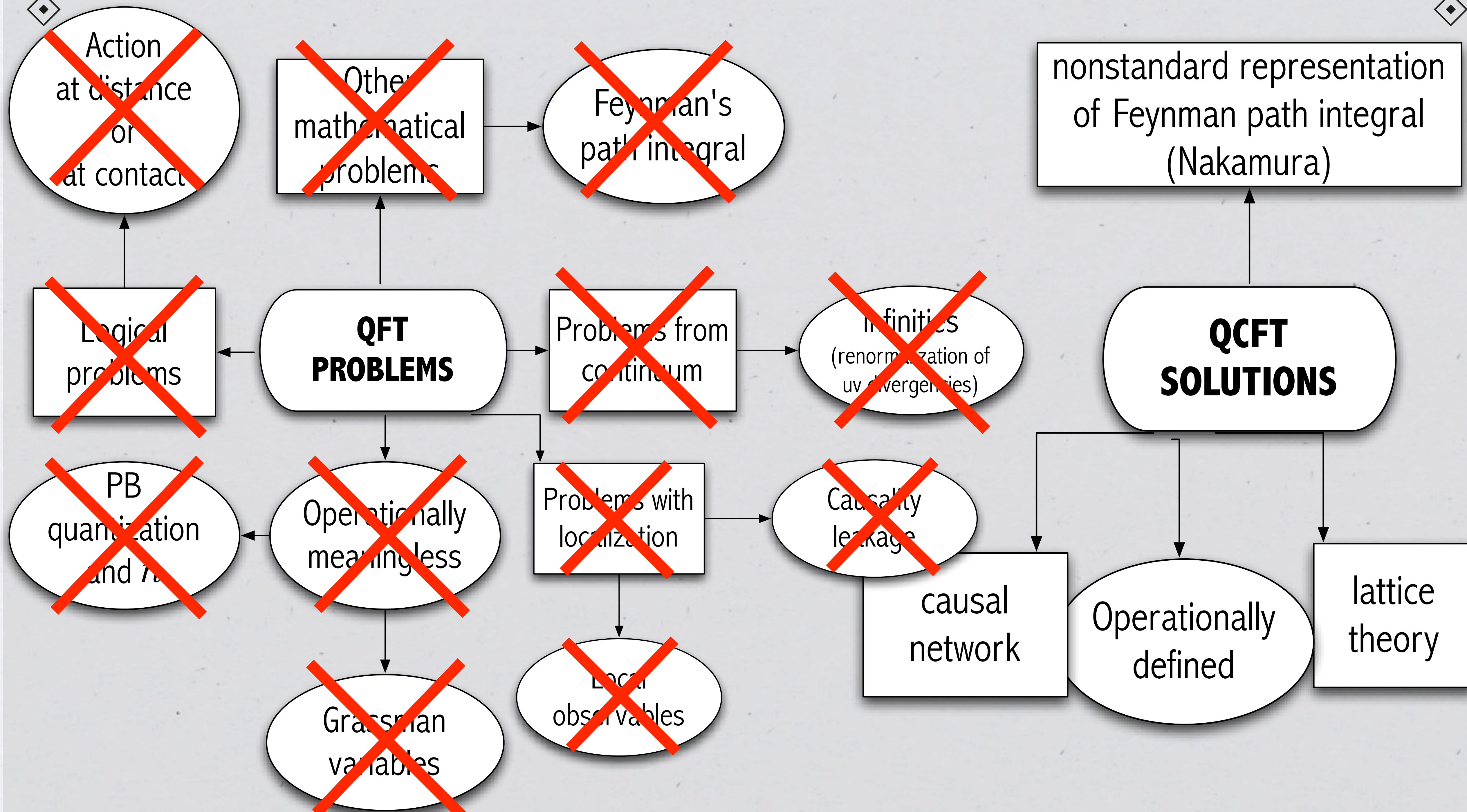
GR from QT?



patterns?

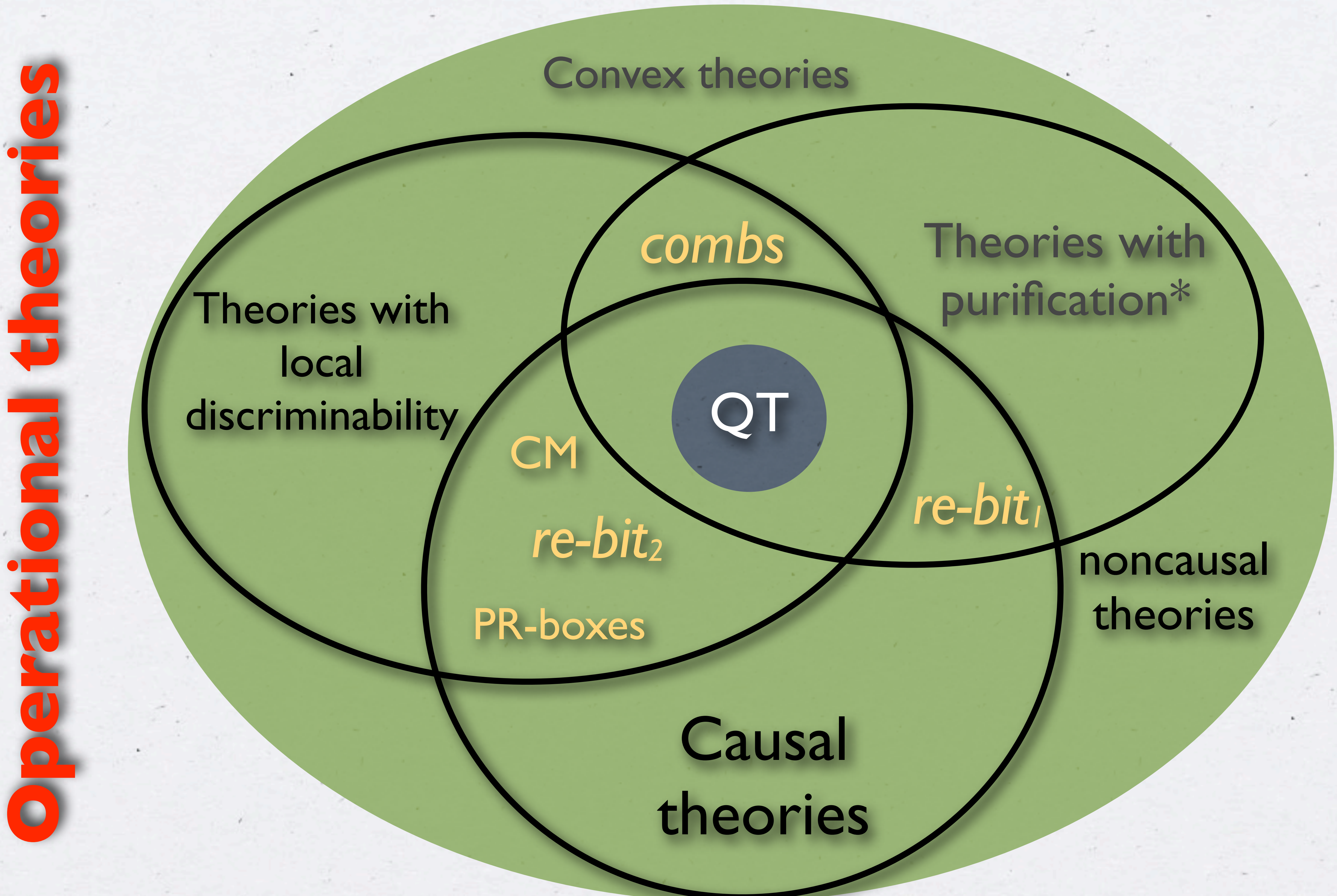


Advantages of QCFT versus QFT



Moreover, you can change the computational engine from QT to super-QT, or even non-causal OpT, without changing the theoretical framework

THE PRINCIPLE OF THE QUANTUMNESS



“Emergent” Physics

- Relativity
- Gravity
- Field Theory
- Quantization rules and ħ
- …

TODO list

- Improve Ichonise and Tamura bound
- Derive Lorentz covariance of field
- Dirac and e.m. field in 3d
- Connect Lagrangian density with a circuit tile
- Derive a 1dim toy (non)abelian gauge theory
- Re-examine microcausality:
    - Fermi, Bose, para-statistics?
- Rederive quantization rules
- Re-derive Feynman path integral via Trotter
- Explore connections with lattice theories
- Rederive GR Einstein's equation
- Explore Penrose spin-networks, Regge calculus, etc.
- Rederive gauge theories
- Write a Theory of ... Quantum Gravity!

Concluding remarks

* QCFT seems to have many advantages versus QFT
* It puts the nose on the foundational problems in QFT
* It is QG-ready
* It's fun! (a good excuse to study more physics)
* It brings Quantum Information to particle physics, GR, and cosmology!